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Estimation of thermal diffusivity of foods using transfer functions

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Abstract

The effective thermal diffusivity of foodstuffs was estimated from time–temperature histories in the geometric center of samples exposed to heating and cooling processes.

Transfer functions methodology was used as an alternative method to estimate the thermal diffusivity assuming that conduction was the main heat transfer mechanism. The samples were characterized as delayed first-order systems with unit gain, dead time (L) and time constant (τ).

The results were compared with those obtained from the f_h value and with results reported in the literature.

Keywords: Thermal diffusivity; Transfer function; Heating; Cooling; Potato; Meat

1. Introduction

Heat transfer is one of the most important physical phenomena that occur during production and food processing. Cooking, blanching, frying, retorting as well as cooling, among others, are processes that are used not only to modify food characteristics (Tijskens, Schijvens, & Biekman, 2001; Wang & Sun, 2001) but also to kill or retard spoilage growth bacteria (Schirra, D'hallewin, Ben-Yehoshua, & Fallik, 2000). Thereby, the selected thermal process must ensure the microbiological safety and the quality attributes of the food product to comply with the consumer demand (Nott & Hall, 1999).

When conduction is the main mechanism controlling heat transfer, the mathematical expression describing the basis of this mechanism in the unsteady state is the Fourier's equation:

$$\rho C_p \frac{\partial T}{\partial t} = \nabla(k \nabla T), \quad (1)$$

where T is the temperature, t is the time, k is the thermal conductivity, C_p is the heat capacity, ρ is the density of the food product and ∇ the differential operator.

Assuming that the material is isotropic and k is constant (independent of temperature), in terms of cylindrical co-ordinates Eq. (1) can be written as

$$\frac{\partial T}{\partial t} = \alpha \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial t} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right], \quad (2)$$

$$\alpha = \frac{k}{\rho C_p}, \quad (3)$$

where α is the thermal diffusivity that controls the heating front in conductive foodstuffs (Holdsworth, 1997) and r, θ and z are the spatial co-ordinates.

A solution of Eq. (2) in terms of time, location and thermal diffusivity requires initial and boundary conditions. The analytical solutions of Eq. (1) for simple geometries are reported in the literature, in particular,

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for an infinite slab and an infinite cylinder (Singh, 1992). For a finite cylinder, the solution can be approximated using Newman's rule (Holdsworth, 1997).

The knowledge of the thermal diffusivity of foods is important to design and optimize processes in which heat transfer is involved (Baucour, Cronin, & Stynes, 2003). Several authors have reported effective thermal diffusivity of foodstuffs as functions of shape, size and thermal properties using heat process data. The log method, the analytical solutions of Eq. (1) and correlations and/or estimation of the property via measurement of thermal conductivity, density and heat capacity are the methodologies usually used (Jaramillo-Flores & Hernandez-Sanchez, 2000; Carciofi, Faistel, Aragao, & Laurindo, 2002; Markowski, Bialobrzewski, Cierach, & Paulo, 2004).

Some authors (Salvadori, Mascheroni, Sanz, & Domínguez Alonso, 1994; Marquez, De Michelis, Salvadori, & Mascheroni, 1998) have applied transfer functions with discrete variables in the z-domain to predict temperature profiles in food products during thermal process that do not involve the specification of the thermal diffusivity.

Nevertheless, the application of transfer functions with continuous variables in the s-domain (Laplace transform) allows a simple link of the defining parameters with the thermal diffusivity.

The purpose of the present work was to estimate the thermal diffusivity of conductive foods from the parameters of their transfer functions when they are assimilated to delayed first-order systems. This approach would present the following advantages: (a) it takes into consideration the whole thermal history and not just the range of temperatures for which the semi-log plot of temperature versus time is linear; (b) it makes use of a simple mathematical model and (c) it may be used with other forcing functions in the temperature of the surroundings, such as pulse, ramp or sinusoidal.

2. Materials and methods

2.1. Materials preparation

The following three different materials were used in the experiments: polyamide cylinder (height 0.10 m; diameter 0.060 m); potatoes cylinders cut from fresh, hand peeled potatoes (height 0.050 m; diameter 0.04 m) and minced beef packed in metal cans (height 0.075 m; diameter 0.065 m). The minced meat was previously processed to obtain a homogeneous puree using a homogenizator (Minipymer, Braun, Argentina) and the cans were completely filled with this preparation.

The samples were subjected to step-like changes in the surrounding temperature by moving them from one well-stirred water-bath to another (M 911 Coleparmer, Germany). The temperatures in the geometrical center of the samples were registered with a temperature data logger (Testo GMBH & Co. Testo - Str. 1, D-79853 Lenzkirtch, Germany).

The polyamide and beef samples were changed from a bath at 30 °C to a bath at 70 °C and the potatoes samples from a bath at 30 °C to a bath at 5 °C.

Three experimental runs were performed with each of the assayed materials.

2.2. Estimation of the thermal diffusivity with transfer functions

The transfer function of a system in the Laplace transform domain is defined as the ratio of output variable $y(s)$ to the input variable $x(s)$ (Stephanopoulos, 1984). The transfer function for a delayed first-order system with unit gain is

$$\frac{y(s)}{x(s)} = \frac{e^{-Ls}}{\tau s + 1}, \tag{4}$$

where L is the dead time and τ the first-order lag.

For the purposes of this work the input variable is the ambient temperature and the output variable is the temperature in the geometric center of the sample. Therefore, Eq. (4) can be written as

$$\frac{T_c(s)}{T_a(s)} = \frac{e^{-Ls}}{\tau s + 1}. \tag{5}$$

Since the ambient temperature undergoes a sudden step-like change, the Laplace transform of T_a is A/s, where A is the magnitude of the step change. The Laplace transform of T_c becomes

$$T_c(s) = \frac{A}{s}\frac{e^{-Ls}}{(\tau s + 1)}. \tag{6}$$

By the inversion of Eq. (6) the evolution of T_c in the time domain is obtained:

$$T_c(t) = T_0, \quad t < L, \tag{7}$$

$$T_c(t) = T_0 + (T_f - T_0)(1 - e^{-(t-L)/\tau}), \quad t \geqslant L \tag{8}$$

or

$$\theta = e^{-(t-L)/\tau}, \tag{9}$$

where T_0 is the temperature of the source bath, T_f is the temperature of the destination bath, t is the time elapsed since the sample is moved from one bath to another and θ is the dimensionless temperature $(T_f - T_c)/(T_f - T_0)$.

Classically, the log method is used to represent the thermal history in the critical point (Magee & Bransburg, 1995). Parameters f and j are used in food thermal process analysis when constant boundary conditions are assumed. j is known as the lag factor, since it measures the lag in establishing a uniform heating rate and f is a measure of the rate of heating or cooling.