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An efficient distributed max-flow algorithm for Wireless Sensor Networks

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ABSTRACT

The max-flow problem (MFP) is one of the most explored problems in the area of combinatorial optimization and it has a wide variety of potential applications in computer networks, especially in Wireless Sensor Networks (WSNs). In this paper, we propose a WSN-specific solution to MFP based on the well-known push-relabel method. In our solution we provide several techniques and heuristics to implement asynchronous communication, reduce message complexity, support adaptability, and satisfy soft real-time requirements. Because these heuristics are independent, we can adjust the algorithm by applying some heuristics and ignoring the others, according to the application requirements. Both theoretical analysis and simulation results show that the proposed algorithm makes a significant improvement in the case of message and time complexities in comparison with the best existing algorithms.

1. Introduction

Due to the recent advances in pervasive and ubiquitous computing, manufacturing of low-cost, tiny and unattended sensors has become feasible. These sensors are capable of measuring specific values from the environmental events and then transmit these values to a specified sink. Wireless Sensor Network (WSN) is a network consisting of a large number of such autonomous sensor nodes which cooperate together to reach some high-level purposes such as surveillance, tracking and monitoring.

The max-flow problem (MFP) is a well-known combinatorial optimization problem in which we wish to send as much flow as possible in a capacitated network between a source node s and a sink node t, without exceeding the capacity of any link. MFP arises in a wide variety of applications and in several forms, not only in the mathematics, management and engineering but also in the specific applications in WSN literature (Song et al., 2009; Bogliolo et al., 2011; Kalpakis, 2010; Al-Kofahi and Kamal, 2009; Ohara et al., 2009; Valero et al., 2012; Dandapat et al., 2010; Yantao et al., 2006).

Several algorithms have been introduced to find the max-flow value which most of them can be categorized into two main categories:

- *Augmenting path algorithms* proceed by identifying augmenting paths which are paths capable of increasing the total net-flow. Then they augment flows on these paths until the network contains no more augmenting path.
- *Push-relabel algorithms*, in which source saturates its outgoing capacities that causes to come out excess flows at neighbors of the source. Push-relabel algorithms gradually mitigate the excess flows by sending some flows from the excess nodes toward the sink, if it is possible, or send back toward the source.

Early proposed solutions for MFP were based on the augmenting path method and the concept of residual networks. The general algorithm of Ford–Fulkerson finds max-flow in $O(mnU)$ time in which U is the maximum link capacity, m depicts the number of links and n is the number of nodes (Ford and Fulkerson, 1956). In Edmonds and Karp (1972) an algorithm is proposed based on this method which terminates in $O(nm^2)$ time. A blocking flow algorithm is designed which finds the max-flow in $O(mn^2)$ time (Dinic, 1973). Since then various algorithms have been proposed based on this method but to our knowledge none of them are distributed and so they are not implementable in WSNs.

On the other hand push-relabel algorithms work locally and they have potential to become a distributed and asynchronous algorithm by some modifications (Goldberg and Tarjan, 1988). Several solutions have been published based on the push-relabel

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method. Authors of Ghosh et al. (1997) have presented a self-stabilizing algorithm which finds the max-flow in almost n^2 messages in the average-case, but their method produces exponential messages in the worst-case. In Pham et al. (2005), based on the push-relabel method (Goldberg and Tarjan, 1988), a two-phase distributed algorithm has been proposed, which is asynchronous and adaptive. This algorithm finds the max-flow in $O(n^2)$ time, using $O(n^2m)$ messages, and leverages depth-first-search that is not efficient in the large-scale networks. In Avestimehr et al. (2011) the channel model and the way of deriving link capacities in wireless networks are investigated.

Recently, Orlin (2013) proposed the fastest strongly polynomial time algorithm which terminates in $O(mn)$ time and even $O(n^2/\log n)$ for the sparse networks, resolving a long standing conjecture. This algorithm which does not place in the above categories beats the fastest existing available algorithms in the literature (Goldberg and Rao, 1998; Cheriyan et al., 1996; Cherkassky and Goldberg, 1997; Chandran and Hochbaum, 2009) but it is a centralized approach like most of the other existing solutions.

There are several parallel algorithms (Shiloach and Vishkin, 1982; Anderson and Setubal, 1992; Bader and Sachdeva, 2005; Caragea and Vishkin, 2011) as well as several GPU max-flow implementations (Hussein et al., 2007; Vineet and Narayanan, 2008), unfortunately none of them could be implemented in distributed wireless ad-hoc and sensor networks. To our knowledge, existing asynchronous distributed solutions which have the potential of being implemented in WSNs are based on push-relabel algorithm (Goldberg and Tarjan, 1988; Pham et al., 2005). Therefore, the proposed algorithm is the first WSN-specific algorithm to solve MFP that is asynchronous, lightweight, adaptive and incremental. We have proposed some raw ideas previously in Homayounnejad et al. (2011a, 2011b). In this paper, we enhance and integrate them, and give more insight into the solution by transparent examples and scenarios. Moreover, we prove the correctness of some heuristics, and we analyze the complexities of some other proposed heuristics.

WSNs are distributed, dynamic, autonomous and resource-constrained networks, hence the proposed heuristics should consider these characteristics. Some applications of max-flow problem in computer networks, especially in WSNs, are life-time maximization (Song et al., 2009), routing (Bogliolo et al., 2011; Ohara et al., 2009; Soorki and Rostami, 2014), aggregation (Kalpakis, 2010), network coding (Al-Kofahi and Kamal, 2009), data redistribution in sink failures (Valero et al., 2012), bandwidth allocation (Dandapat et al., 2010), traffic management (Yantao et al., 2006), etc. As one special case, consider the following routing applications:

1. In the multipath routing, maximizing the number of edge-disjoint paths from some sources to a target could be cast to the max-flow problem in the unit-capacitated network (Ahuja et al., 2014).
2. In energy sustainable sensor networks, as long as the average workload at each node can be sustained by the average power it takes from the environment, the node can keep working for an unlimited amount of time. The power-constrained workload optimization can be modeled and solved by the max-flow problem (Bogliolo et al., 2011).
3. We hope that new QoS-aware routing protocols could be designed based on the heuristics proposed here. Because it tries to maximize the throughput of the network and so it can be combined with the effective-bandwidth theory (Wu and Negi, 2006) and statistical methods (Abdrabou and Zhuang, 2009) to propose a QoS-aware protocol which can take into account delay, reliability and bandwidth, simultaneously.

Despite the various applications of max-flow in WSNs, to our knowledge there is not any dedicated work for implementing a max-flow solution in WSNs. The proposed algorithm and its corresponding heuristics try to fill this large gap and to open a new window to WSN solutions.

The rest of the paper is organized as follows. Section 2 describes the fundamental concepts and the network model. In Section 3, the proposed algorithm and its heuristics are described in detail. The theoretical complexity analysis and the correctness proofs are included in Section 4, and simulation results are presented in Section 5. Finally, Section 6 concludes the paper.

2. The notation and assumptions

A WSN is modeled as a network $G = (V, E)$ whose nodes V are sensors and E is the set of the full-duplex directed communication links. Each sensor of WSN has a unique identifier and has at least one transmitter and one receiver. We assume that the effective transceiving distance of all nodes is equal. Two nodes are neighbors and have a link between them if they are in the transceiving range of each other. Moreover, we assume that a mechanism exists for neighbor discovering and an underlying protocol to deliver a packet from a node to its one-hop neighbor, properly. Every link $(i,j)\in E$ is augmented with a nonnegative capacity c_{ij} which restricts the flow on this link (f_{ij}). Solving MFP is to find the max-flow from the source node s to the sink node t that satisfies some constraints, formulated as follows:

$$\text{maximize } |f| \tag{1}$$

subject to

$$\sum f_{ij} - \sum f_{ji} = \begin{cases} |f| & \text{for } i=s \\ 0 & \text{for all } i\in V-\{s,t\} \\ -|f| & \text{for } i=t \end{cases} \quad \text{(Mass balance constraint)} \tag{1 − a}$$

$$0\le f_{ij}\le c_{ij} \quad \text{for all } (i,j)\in E \quad \text{(Capacity constraint)} \tag{1 − b}$$

The value of a flow f is defined as $|f| = \sum_{v\in V} f(s, v)$ that is, the total flow out of the source. For example, as one potential application, the QoS-aware routing could be proposed based on MFP. Bandwidth of the links can be modeled by link capacities, and therefore the max-flow is maximum utilization of wireless bandwidths.

Definition 1 (Residual Network (G_f)). *Residual capacity* is the maximum additional flow that can be sent from node i to node j using the links (i, j) and (j,i) and could be computed by means of capacities and flow values as $residual[u, v] = c[u, v] - f[u, v]$. The *residual network* is the network consisting links that have positive residual capacities.

Definition 2 (Height function (h)). A function $h: V \to N$ is a height function if $h[s]=n$, $h[t]=0$, and $h[u] \le h[v] + 1$ for every residual link $(u, v)\in G_f$. Algorithm pushes flow downhill, which is from the higher node to the lower node.

Definition 3 (Generic push-relabel algorithm (GPR)). Algorithm 1 is the sequential push-relabel (Goldberg and Tarjan, 1988), which uses push and relabel operations and nodes could maintain a positive excess value during the running of the algorithm. In this algorithm, the push operation tries to send the excess flow to lower-height neighbors, if any exists. Otherwise, the relabel operation would be invoked to elevate the height and to make the push operation applicable. Throughout this paper, we