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Fractal dimension of soil fragment mass-size distribution: A critical analysis

Behzad Ghanbarian *, Hugh Daigle

Department of Petroleum and Geosystems Eng., University of Texas at Austin, Austin, TX 78712, United States

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ABSTRACT

In this rapid communication, we address two important issues regarding the calculation of fragment mass-size distribution fractal dimension. We particularly focus on particle-size distribution, as a special case of fragment mass-size distribution, and demonstrate that the arithmetic mean concept frequently used in the literature is not supported. We also show that ignoring lower and upper cutoffs of fractal scaling may significantly alter the mass fractal dimension value. For these purposes, two examples using experiments available in the literature are given, and critical analyses are discussed in detail. We also reanalyze three databases reported in the literature, recalculate the mass fractal dimension, and show that applying the arithmetic mean concept and/or ignoring the lower and upper cutoffs may result in substantially different fractal dimension values. We note that the lower and upper cutoffs have to be reported in addition to the mass fractal dimension value. We also emphasize that accurate determination of the fractal scaling parameters, such as fractal dimension and lower and upper cutoffs requires precise characterization of the mass-size distribution.

1. Introduction

Natural soils are mixtures of particles (solid grains) and aggregates (porous fragments) that are made up of particles. Aggregates form by building-up (e.g., cementation and cohesion) processes and are destroyed by breaking-down (e.g., tillage) mechanisms. The breaking of soil aggregates into smaller pieces or fragments is called fragmentation, which is a typical phenomenon in nature occurring due to external stresses, such as tillage (Perfect et al., 2002). The fragmentation process depends mainly on the parent materials and the fragmentation type e.g., scale invariant (Perrier and Bird, 2002).

Although various fractal fragmentation models have been proposed in the literature, such as Turcotte (1986) and Perfect et al. (2002), in this study we apply the pore–solid fractal (PSF) model. In this model, three phases exist: pore (P), solid (S), and fractal (F). The fractal phase is the only one iterated in the fractal construction procedure (see Fig. 1). In each iteration, regions previously assigned to the fractal phase are randomly partitioned into P, S, and F; the fractional value assigned to each phase does not change across iterations, and $P + S + F = 1$ (Ghanbarian et al., 2015).

Based on the PSF approach (Perrier et al., 1999), Perrier and Bird (2002) developed a scale-invariant fragmentation model. According to their model, aggregate- and particle-size distributions represent incomplete and complete fragmentation processes, respectively. In the PSF approach framework, the fragmentation fractal dimension is given by (Perrier and Bird, 2002)

$$D_f = \frac{D \log(pF)}{\log(F)} \tag{1}$$

where F is the fractal portion in the PSF model, p is the fragmentation probability (the probability that a PSF unit fragments at each scale), and D is the mass fractal dimension (fractal dimension of particle-size distribution). For incomplete fragmentation, $p < 1$, and thus $D_f < D$. For complete fragmentation, $p = 1$, meaning all aggregates break down into primary particles and therefore the fragmentation fractal dimension $D_f = D$.

Following Perrier and Bird (2002), the cumulative mass of fragments of size R_i and smaller, $M(<R_i)$, is given by

$$\frac{M(<R_i)}{M_t} = \alpha^{D-E}\left(\frac{R_i}{L}\right)^{E-D_f}, \quad R_{min} \le R_i \le R_{max} \tag{2}$$

where $\alpha < 1$ is the scaling factor (the similarity ratio of the partitioning process in the PSF model), E is the Euclidean dimension ($E = 3$ in three dimensions), M_t is the total mass of fragments, L is the initiator size ($R_{max} = \alpha L$, in which R_{max} is the largest fragment size), and R_{min} and R_{max} are the lower and upper cutoffs. Natural porous media, such as soils and rocks showing self-similar behavior, may lose their fractal

* Corresponding author.
E-mail address: ghanbarian@austin.utexas.edu (B. Ghanbarian).

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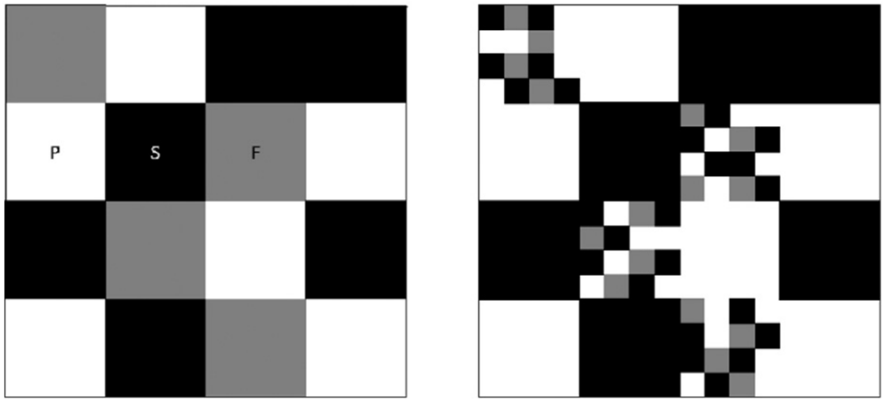

Fig. 1. Two-dimensional random pore–solid-fractal (PSF) models (generator on left and first iteration on right) proposed by Perrier et al. (1999) where $P = 0.375$ (pore phase portion), $S = 0.375$ (solid phase portion), and $F = 0.25$ (fractal phase portion).
After Ghanbarian-Alavijeh (2014).

Table 1
Particle-size distribution of the Ariana soil (Rieu and Sposito, 1991; Perrier and Bird, 2002).

R_i (mm)	$M(<R_i)$ (kg)
1.79	0.10062
1.414	0.08667
1.118	0.06567
0.89	0.05316
0.71	0.04276
0.561	0.03336
0.4	0.02699
0.251	0.01799
0.141	0.01079

properties below and above the lower and upper cutoff scales, as we demonstrate.

The distribution for primary particles results from complete fragmentation ($p = 1$ and $D_f = D$). Therefore, for the particle-size distribution, Eq. (2) reduces to (Perrier and Bird, 2002)

$$\frac{M(<R_i)}{M_t} = \left(\frac{R_i}{R_{max}}\right)^{E-D}, \quad R_{min} \le R_i \le R_{max}. \tag{3}$$

Eq. (3), widely applied to determine the mass fractal dimension D (or fractal dimension of particle-size distribution), is the same model derived by Turcotte (1986) and Tyler and Wheatcraft (1992) using different methodologies. We should emphasize that $M(<R_i)$ in Eq. (3) is the cumulative mass of particles below an *upper limit*, R_i, practically the upper sieve size, and R_{max} is the upper size limit (cutoff) for fractal scaling (Tyler and Wheatcraft, 1992; Bittelli et al., 1999; Millán et al., 2003; Filgueira et al., 2006). In several studies, however, R_{max} has been set equal to the largest measured particle size e.g., 1.5 mm (Bayat et al., 2013), and 2 mm (Peng et al., 2014). We discuss this misconception in the following.

In the literature, another form of Eq. (3) has been frequently applied (see e.g., Su et al., 2004; Liu et al., 2009; Ai et al., 2012; Cao et al., 2013; Xu et al., 2013):

$$\frac{M(<R_i)}{M_t} = \left(\frac{\overline{R}_i}{\overline{R}_{max}}\right)^{E-D} \tag{4}$$

where $\overline{R}_i$ is the mean particle size of the *i*th size class given by the arithmetic mean of the upper and lower sieve sizes, and $\overline{R}_{max}$ is the arithmetic mean size of the largest size class. Strictly speaking, the power-law fractal scaling Eq. (3) including R_i necessarily as the *upper* limit was developed *theoretically*. However, the arithmetic mean concept in Eq. (4) is merely empirical and arbitrary, and Eq. (4) has never been derived mathematically, to the authors' knowledge.

In order to apply Eq. (4) to measured particle-size distribution, in analogous fashion, the mean size of the particles smaller than 2 μm is calculated as 1 μm (see e.g., Su et al., 2004; Zhang et al., 2009; Ai et al., 2012; Cao et al., 2013). This means particles smaller than 2 μm follow the same fractal scaling that particles larger than 2 μm do and thus $R_{min} = 0$. However, Wu et al. (1993) and Bittelli et al. (1999), among many others, indicated that the whole range of the particle-size distribution might be characterized by more than a single fractal regime, which means that R_{min} of each regime must have a finite, nonzero value. As a consequence, setting the mean size of the particles smaller than 2 μm equal to 1 μm is dubious and not supported. In addition, the arithmetic mean concept used in Eq. (4) constitutes an internal

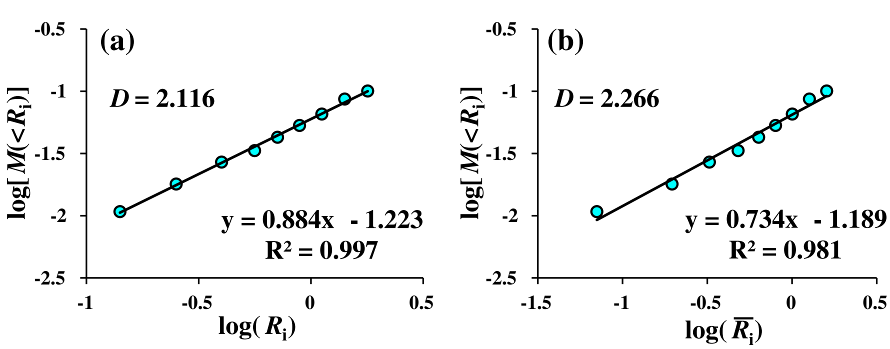

Fig. 2. Determination of the fractal dimension of the particle-size distribution using (a) Eq. (3), and (b) Eq. (4) for the Ariana soil. Fitting Eqs. (3) and (4) to the measured data yielded $D = 2.116$ (2.075, 2.157) and 2.266 (2.176, 2.357), respectively (values in the parentheses are the 95% confidence bounds).