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Universal soil shrinkage curve equation

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ABSTRACT

The soil shrinkage curve is used to relate the volume of soil as it dries with water content. The shape of shrinkage curve depends on the soil type and soil fabric. A shrinkage curve can consists of two to four phases. It is difficult to represent all types of shrinkage curves with a single continuous equation which uses explicit parameters. In this paper, a universal shrinkage curve equation with explicit parameters is proposed which can be used for all types of shrinkage curves. Application of the proposed equation is illustrated with published data. Comparison of the proposed equation with other shrinkage curve equations is made in terms of goodness-of-fit measures with experimental data.

1. Introduction

The shrinkage curve is an important relationship of the soil. The shrinkage curve is usually plotted as void ratio versus water content. The water content on the shrinkage curve can be either gravimetric water content (Fredlund et al., 2002) or moisture ratio (Cornelis et al., 2006a, 2006b). The shape of shrinkage curve differs for different soil types and soil fabrics. Peng and Horn (2013) proposed six types of shrinkage curves (A, B, C, D, E and F) based on the existence of the four shrinkage phases (i.e. zero shrinkage, residual shrinkage, proportional shrinkage and structural shrinkage). The difference in the types of the shrinkage curves relates to the existence of the four shrinkage phases as summarised in Table 1. The shrinkage limit, macropore shrinkage limit, and saturated moisture ratio separate each phase of the shrinkage curve. However, there are some differences regarding the definitions between disciplines. In geotechnical engineering, the point used to separate different phases is usually given by the intersection point between adjacent linear segments while in soil science, the point of convergence between the curvilinear part and the linear segment is used. For example, Fredlund et al. (2002) and Holtz et al. (2011) defined the shrinkage limit as the intersection point between the loading line and the zero shrinkage line (shown as w_2 in Fig. 1). This definition may not give the true shrinkage limit (SL) as further decrease of water content will cause additional shrinkage. This point will be known as apparent shrinkage limit (SL′) in this paper. On the other hand, Cornelis et al. (2006a, 2006b) defined shrinkage limit as the point of convergence between the curvilinear segment of the residual shrinkage and the zero shrinkage line where there is no further decrease in void ratio below this water content (shown as w_{2-} in Fig. 1). This point is the true shrinkage limit (SL). The true shrinkage limit is difficult to determine as it requires continuous measurement. In this paper, every intersection points and the points of convergence between the linear and the curvilinear segments of the shrinkage curve are defined as shown in Fig. 1.

The shrinkage curve can be modelled as a combination of linear and curvilinear segments (Braudeau et al., 1999). Using such an approach, a general shrinkage curve consists of n-linear segments where each segment i with slope m_i represents a different phase of the shrinkage curve (Fig. 1) and the curvilinear segment between two linear segments being a curve with curvature k_i. Types A and B shrinkage curves can be considered as a 4-linear segment shrinkage curve (Fig. 1a), types C and D can be considered as a 3-linear segment shrinkage curve (Fig. 1b), while types E and F can be considered as a 2-linear segment shrinkage curve (Fig. 1c).

Parameters in the shrinkage curve equation can be categorised into explicit or implicit parameters. An explicit parameter is one which represents a single property of the curve (e.g., the slope or intercept of the curve) whereas an implicit parameter is one which affects several properties of the curve. Several advantages in using explicit parameters in a shrinkage curve equation are as follows:

1. Explicit parameters can be directly obtained from the shrinkage curve data.
2. The values of explicit parameters do not depend on the form of equation and any error minimization criterion in determining their values.
3. As explicit parameter is unique, it can therefore be investigated separately, for example in correlating the parameter with other soil properties.

Although explicit parameters of the shrinkage curve can be easily obtained, it is not so easy to describe all types of shrinkage curve

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Table 1
Classification of shrinkage curves based on the four shrinkage phases.

Type	Structural shrinkage	Proportional shrinkage	Residual shrinkage	Zero shrinkage
A	Yes	Yes	Yes	Yes
B	Yes	Yes	Yes	No
C	Yes	Yes	No	No
D	No	Yes	No	No
E	No	Yes	Yes	No
F	No	Yes	Yes	Yes

using a single continuous equation with the explicit parameters. It is more common to use linear equation to describe the linear segments and another equation to describe the curvilinear segments of the shrinkage curve. Some of the extant shrinkage curve equations are summarised in Table 2. Chertkov (2000, 2003) used a combination of linear and polynomial equations to describe the shrinkage curve. Kim et al. (1992), Cornelis et al. (2006a) and Braudeau et al. (1999) used a combination of linear and exponential functions to describe the shrinkage curve. Giraldez et al. (1983) and Tariq and Durnford (1993) used a polynomial function to describe the shrinkage curve. Fredlund et al. (2002) and Olsen and Haugen (1998) used a hyperbola function, while McGarry and Malafant (1987), and Peng and Horn (2007) used a logistic function to describe the shrinkage curve. Cornelis et al. (2006a) compared eight shrinkage curve equations and concluded that the McGarry and Malafant (1987), Braudeau et al. (1999), modified Chertkov (2000, 2003) by Cornelis et al. (2006a), modified Groenevelt and Grant (2001, 2002) by Cornelis et al. (2006b) equations performed the best in terms of root mean square error (RMSE), coefficient of correlation (R^2) and the Akaike Information Criterion (AIC) with some qualifications. The McGarry and Malafant (1987) equations do not describe the zero shrinkage phase. The original Chertkov (2000, 2003) equations are for clay pastes. Although the modified Groenevelt and Grant (2001, 2002) equation (Cornelis et al., 2006b) uses only one single equation to describe the complete shrinkage curve, it uses implicit parameters. An error minimization criterion is used to obtain the implicit parameters.

In this paper, a single continuous equation that uses explicit parameters to describe a shrinkage curve is developed to describe any type of shrinkage curve.

2. Development of a Universal Shrinkage Curve Equation

Fig. 1 shows shrinkage curves with 2, 3 and 4-linear segments with a smooth transition between the segments where m_i is the slope of the linear segment i, k_i describes the curvature of the curvilinear segment and x_i is the intersection point between segment i and segment (i − 1), respectively. The x-axis can be either gravimetric water content (w) or moisture ratio (υ). The slope between segment i and segment i − 1 is given as:

$$m = m_{i-1} + H_i(x, x_i)(m_i - m_{i-1}) \tag{1}$$

where $H_i(x,x_i)$ is a Heaviside function to describe the transition of slope from the slope of segment i − 1 to the slope of segment i. The simplest form of such a function is the Heaviside or unit step function where

$$H(x, x_i) = \begin{cases} 0 & x < x_i \\ 1 & x \geq x_i \end{cases}. \tag{2}$$

When a smooth transition is desired, the approximate value of $H(x, x_i, k_i)$ can be given by:

$$H(x, x_i, k_i) \approx \frac{1}{2} + \frac{1}{2}\tanh k_i(x - x_i) \tag{3}$$

where k_i describes the curvature at the intersection point x_i. When k_i approaches infinity, Eq. (3) tends to Eq. (2). The advantage of using Eq. (3) is that it can be integrated, as well as differentiated. The integral of Eq. (3) is known as the Ramp function $R(x, x_i, k_i)$ and is given by:

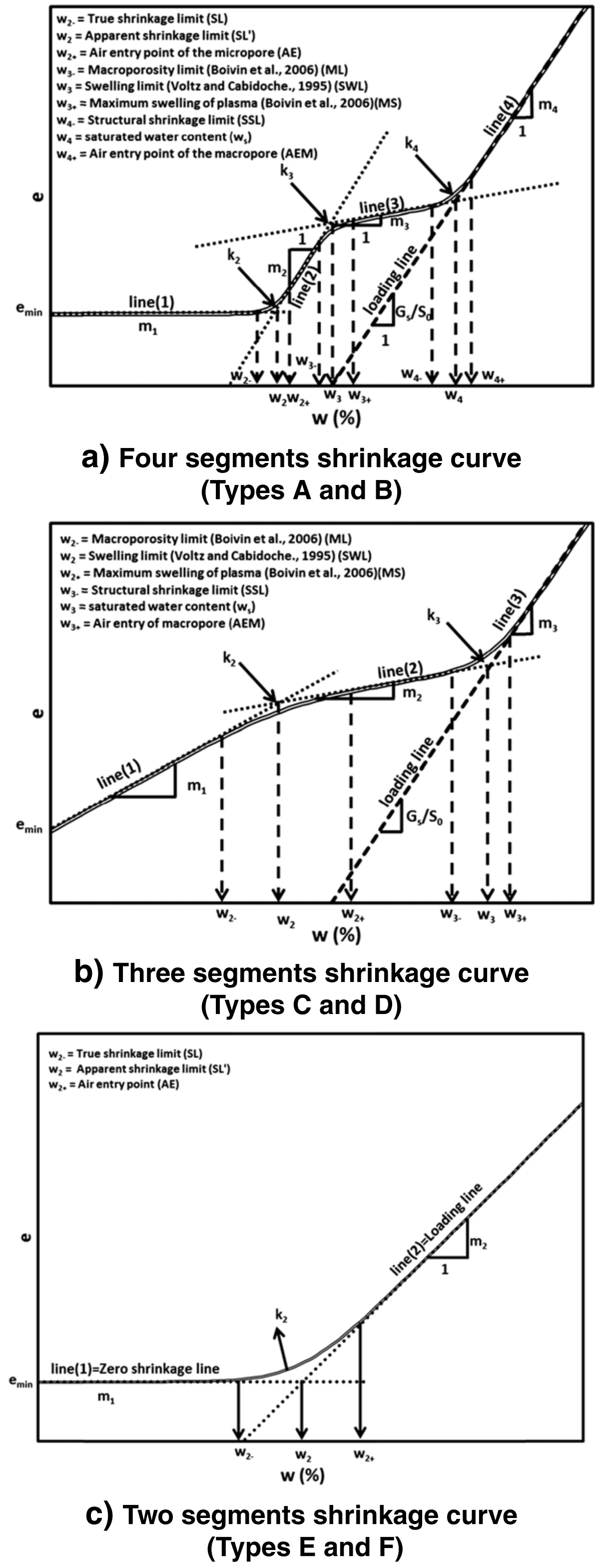

Fig. 1. Shrinkage curve phases and boundaries.

$$R(x, x_i, k_i) = \int_{x=x_0}^{x=x} H(x, x_i, k_i) = \frac{1}{2}\left\langle (x - x_0) + \frac{1}{k_i}\ln\left\{\frac{\cosh[k_i(x - x_i)]}{\cosh[k_i(x_i - x_0)]}\right\}\right\rangle. \tag{4}$$