



A stochastic geometric model for continuous local trends in soil variation

R.M. Lark*

British Geological Survey, Keyworth, Nottinghamshire NG12 5GG, UK

ARTICLE INFO

Article history:

Received 13 February 2012

Received in revised form 30 May 2012

Accepted 11 June 2012

Available online 26 September 2012

Keywords:

Linear mixed model

Stochastic geometry

Voronoi tessellation

Multiple point geostatistics

Topofunction

ABSTRACT

This paper develops and demonstrates a model of stochastic spatial variation. It is proposed that this model may represent soil variability according to a particular mode under which the soil varies continuously, showing short-range lateral trends induced by local effects of the factors of soil formation which vary across the region of interest in an unpredictable way. The trends in soil variation are therefore only apparent locally, and the soil variation at regional scale appears random. Such variation might be expected in a landscape where the soil varies along topographic catenas which repeat across the region in response to a drainage pattern which is not entirely regular in spacing or orientation, and is therefore unpredictable. The Continuous Local Trend (CLT) mode of soil variation may also be expected where gradients of soil properties are induced around individual plants, or plant roots.

In the stochastic model the local trend is assumed to be described by a function of distance to the nearest event in a realisation of a random spatial point process. A model is developed here in which the point process shows complete spatial randomness, so it is called the Poisson Continuous Local Trend (PCLT) model. The covariance function for the PCLT with a general distance function is developed and some hypothetical examples are shown, including one in which the variogram of a soil property is inferred by using a published topofunction. The PCLT model is then fitted to the empirical variogram of some data on soil water content in a gently undulating clay landscape, and the multiple point statistics of the PCLT model for these data are compared with those of a corresponding multivariate normal model.

© 2012 Natural Environment Research Council (NERC). Published by Elsevier BV. All Rights Reserved.

1. Introduction

Geostatisticians use mixed models to analyse and predict soil properties. In these models some of the soil variation is accounted for by fixed effects, continuous covariates or categorical factors, and the remaining variation is modelled as random effects, including a spatially correlated component (Lark et al., 2006). Typically our knowledge of soil processes is put to use by selection of appropriate fixed effects for such models. The random effects account for the soil variation that we cannot explain in terms of fixed effects. Either no fixed effects can be formulated, because of the complexity of the origins of the soil variation and its dependence on contingent events in the prehistory of the landscape (Webster, 2000), or appropriate covariates are not measured at the scale of interest in the region under study.

The spatial correlation of the random effects is modelled by a covariance function typically selected from a set of authorised functions with convenient mathematical properties (Webster and Oliver, 2007). However, covariance models for the random effects would ideally be selected because they represent the processes that cause the variation. One advantage of such an approach would be that prior distributions for the covariance parameters could be specified from scientific knowledge and understanding of the underlying processes. These prior models could

then be used to improve the efficiency of sampling (Marchant and Lark, 2006).

The relationship between the form of the covariance function and the underlying physical processes is well established for diffusion (Whittle, 1954; 1962) and for variables in branches of the earth sciences including hydrology (Kolvos et al., 2004) and geophysics (Chilès and Delfiner, 1999), but we might reasonably observe that in most cases the factors underlying soil variation are too complex to allow a straightforward inference from process understanding to the form of the covariance function. However, we might identify a model of random variation in space that represents a general mode of soil variation that we can expect to encounter in particular conditions.

By a mode of soil variation is meant a simple and generalisable rule that captures how the effect of a factor of soil formation varies laterally. The mode of variation for a variable is a basis for prediction of features of its statistical distribution (e.g. Allègre and Lewin, 1995) and for decisions such as the selection of a transformation or model. For example, if soil variation is associated with microtopography in a landscape which shows pronounced and regular periodicity (e.g. ridge and furrow), then we might call the expected mode of variation *periodic*, and expect to see a variogram with a regular fluctuation. Webster and Oliver (2007) note that apparent fluctuations in the empirical variogram can be artefacts, arising, for example, from strongly clustered sampling, and advise against the routine selection of periodic variograms models just because they fit. Pedological knowledge that a variable arises from a periodic

* Tel.: +44 115 9363026.

E-mail address: mlark@nerc.ac.uk.

mode of variation gives us confidence both to select a variogram model with a periodic component and to interpret the wavelength of the fluctuation in the variogram as real information about the underlying mode of variation (its wavelength) and the soil-forming factors that underlie it.

Lark (2009) considered another mode of soil variation where the factors of soil formation operate within *discrete domains* (different geological units, agricultural fields, catchments etc.) The Poisson Voronoi Tessellation (PVT) model was proposed for random variation of soil according to this mode, based on the partition of space into Dirichlet tiles around seed points drawn from a Poisson spatial point process. The model fitted well to the empirical variograms of soil properties measured at a range of scales. Lark (2010) showed that the PVT model was a more plausible model of the variation in several soil data sets than was an alternative multivariate normal model. However, it is clear that a model based on discrete domains will not be universally appropriate for the random variation of the soil. It is necessary to develop a wider range of random models for other modes of soil variation.

In this paper I propose a random model for soil variation that exhibits *continuous local trends* (CLT). This mode of soil variation can be exemplified at disparate spatial scales. For example, gradients of soil properties may be induced around individual plants (Pérez, 1995) or individual rhizospheres (Youssef and Chino, 1989). Gradients of soil properties have also been reported from the centre to the margins of the polygons in patterned ground (Barrett et al., 2004). Such variation is continuous (there are no step changes in the soil property), and is characterised by lateral trends. However, the trend is not global (at the scale of the whole region of interest) but rather is local induced by an underlying process such as the distribution of plants, roots or periglacial polygons whose distribution is not predictable at a global scale. The local trends therefore form a repeating pattern across the region, which cannot be regarded as a simple deterministic function (unlike a global trend across the region), and may, in the absence of an appropriate covariate (such as a remote sensor image of patterned ground) be consigned to the random effects of a mixed model.

The CLT mode of soil variation is exemplified at landscape scale by certain forms of catenary variation. The concept of the catena was introduced by Milne (1936) to facilitate soil survey in East Africa. Milne's catenas represent a pattern of soil variation across a valley from drainage line to interfluvium. Variation along a catena may be continuous, or abrupt: for example at the transition from woodland to grassland at the margin of the dambo which occupies the bottom of the catena described by Webster (1965). In a catenary landscape soil varies predictably across a valley from one interfluvium to the next, but across a region this sequence repeats, constituting a pattern. Milne delineated map units within which a characteristic catenary pattern of variation could be discerned. One might, as Webster (2000) observed, regard the variation of a soil property at locations within such a unit as random because of the unpredictability of the drainage pattern. One such landscape is the Eldama land system in Western Kenya, as surveyed by Scott et al. (1971). A block diagram of this land system is shown in Fig. 1a. In the mixed model context one might assign this variation to fixed effects if it can be represented by covariates, perhaps drawn from a digital elevation model, or otherwise to random effects. The CLT mode of variation would be exemplified by a repeating catenary pattern which can be represented by a continuous topofunction (Yaalon, 1975) such as those proposed for various landscapes by Walker (1966), Ruhe (1969), Walker and Ruhe (1968), Walker et al. (1968) and Kleiss (1970). Continuous local trends, associated with topography are also predicted by pedogenetic models (e.g. Rosenbloom et al., 2001). In those landscapes where the drainage is strongly oriented in one direction the CLT mode of variation is essentially one-dimensional (across the drainage line), this is illustrated by the Lolimo land system in the survey of Western Kenya by Scott et al. (1971), shown in Fig. 1b. A two-dimensional mode of variation could be envisaged in circumstances where the local direction of the drainage line is unpredictable for a randomly located site in the region.

In this paper I propose a stochastic model for the CLT mode of soil variation. In this model it is assumed that local trends are induced by the events in a realisation of a random spatial point process (which could, for example, correspond to positions of individual plants in the example of CLT variation presented by Pérez (1995)). The value of the CLT process at any location depends on the distance to the nearest event from the underlying point process. In this paper I assume complete spatial randomness of the point process, which induces a Poisson CLT (PCLT). In the remainder of this paper I derive this model in more detail and show the form of the variogram for a number of hypothetical instances. I then fit the PCLT model to the empirical variogram of some data on the water content of soil in an undulating clay landscape in eastern England.

2. Theory

In this paper I propose a Poisson CLT (PCLT) model of random variation in which the value of the variable at some location is a function of the distance to the nearest event from a Poisson spatial point process with specified intensity. In this section I develop this model and derive the variogram function for it.

2.1. Notation

Let $\mathbf{s}$ be an arbitrary location in our region of interest which is a d -dimensional real subspace $R \subset \mathbb{R}^d$. Let $\mathbf{h} \in \mathbb{R}^d$ be a 'structuring element', i.e. a vector of unit norm and arbitrary direction (on the assumption of isotropy when $d > 1$), and let r be a lag distance.

Let Ψ be a point process in $\mathbb{R}^d$. This is a random process and a realisation of it in $\mathbb{R}^d$, ψ , is a set of points with random positions, $\mathbf{x}_i \in \mathbb{R}^d$, $i \geq 0$. Denote by S some subspace of our d -dimensional space, $S \subset \mathbb{R}^d$. By $|S|$ is denoted the Lebesgue measure of S , (that is the length in one dimension, area in two dimensions, volume in three dimensions etc.). Let $\psi(S)$ denote a random variable which is the number of events of the point process in S . The intensity of the spatial point process is λ such that

$$E[\psi(S)] = \lambda |S|, \quad (1)$$

where $E[\cdot]$ denotes the expectation of the term in square brackets. The distribution of $\psi(S)$ is denoted by $P\{\psi(S)\}$, which is a Poisson distribution if the events of the process are completely spatially random and independent. I assume here that Ψ is a stationary random process (homogeneous Poisson) so that statistics such as the intensity are invariant under a translation in space.

A Poisson Voronoi tessellation of S , denoted by T , is the partition of S into non-overlapping space-filling cells which depend on ψ . The i th cell of T , $C_i(\psi)$, contains the i th point in ψ and only the i th point because $C_i(\psi)$ is defined as the set of all points in S which are closer to the i th point in ψ than to any other point in ψ . The boundary of the i th cell is denoted ∂C_i , and the tessellation T is defined uniquely by its skeleton, the union of all the cell boundaries $\partial T = \bigcup_{i \geq 0} (\partial C_i)$.

For any vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^d$, $B[\mathbf{u}, \|\mathbf{v}\|] \subset \mathbb{R}^d$ denotes the closed d -ball of radius $\|\mathbf{v}\|$, such that $\forall \mathbf{s}, \mathbf{s} \in B[\mathbf{u}, \|\mathbf{v}\|]$ if and only if $\|\mathbf{s} - \mathbf{u}\| \leq \|\mathbf{v}\|$.

By $\mathcal{U}(B[\mathbf{u}, \|\mathbf{v}\|], B[\mathbf{u}', \|\mathbf{v}'\|]) \subset \mathbb{R}^d$ is denoted the union of the two balls that are arguments of the expression.

2.2. The Poisson continuous local trend (PCLT) model and its variogram

Consider an arbitrary point $\mathbf{s} \in S \subset \mathbb{R}^d$, where S contains points of a realisation ψ of a Poisson process with intensity λ . We denote by $K(\mathbf{s})$ a random variable

$$K(\mathbf{s}) = \min\{\|\mathbf{s} - \mathbf{x}_i\|, \forall i \geq 0\}, \quad (2)$$

that is to say, it is the distance from $\mathbf{s}$ to its nearest neighbour in ψ . Under the PCLT model for a variable Z , it is a random function

$$Z(\mathbf{s}) = \mathcal{D}(K(\mathbf{s})), \quad (3)$$

Download English Version:

<https://daneshyari.com/en/article/4573640>

Download Persian Version:

<https://daneshyari.com/article/4573640>

[Daneshyari.com](https://daneshyari.com)