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MIMO two-way relay channel with superposition coding and imperfect channel estimation

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ABSTRACT

This paper deals with the superposition coding (SPC) scheme in multiple-input multiple-output two-way relay channels subject to imperfect channel estimation. In this scenario, two multiple antenna terminals, which are unable to communicate directly, exchange information with each other via a multiple antenna relay. We determine the impact of the channel estimation error degradation on the achievable rate region for two main SPC techniques: (a) SPC without channel state information (CSI) at the users, (b) SPC with an imperfect CSI at the users where a waterfilling power allocation is employed. We demonstrate that imperfect CSI significantly improves the achievable rate at low signal-to-noise ratios (SNRs) while it becomes less critical at high SNRs. In addition, a SPC power allocation technique that incorporates the average channel statistics and does not require any instantaneous CSI is also investigated. We show how the available power is split between the two bi-directional (superimposed) data flows in order to maximize the system performance and to support fairness as well as to maximize the achievable sum-rate.

1. Introduction

The two-way relay channel is an important information theoretic network structure (Shannon, 1961). It consists of two users that are unable to communicate directly so instead they establish communication via a shared relay node. This scenario characterizes networks with central controllers as well as ad hoc networks with limited relaying resources. In two-way relay channels the two terminals exchange messages with each other via the relay. The objective of two way relay protocol design is to maximize the spectral efficiency of the two communication links that are formed between the two links.

Recently there is a lot of interest in the design of efficient cooperative protocols for two-way relay channels. The proposed schemes can be divided into two main categories based on the number of the required time phases: (a) In the two phase protocol, called multiple Access Broadcast (MABC) protocol (Kim et al., 2008), both users simultaneously transmit their data to the relay during the first phase and then only the relay transmits during the second phase, while (b) in the three phase protocol the two users sequentially transmit to the relay followed by a transmission from the relay (Saleh et al., 2009). Furthermore, each protocol category can be combined with any relaying strategy resulting in numerous two-way protocols with different complexities and performance. In Kim et al. (2011) the authors highlight the most significant two-way relaying protocols and characterize their achievable rate region from an information theoretic point of view. An interesting MABC two-way scheme that employs superposition coding (SPC) at the relay node has been proposed in Hammerstrom et al. (2007), where the authors analyze its capacity performance for a general Multiple-input multiple-output (MIMO) two-way relay channel with different channel-side information (CSI) requirements. The use of SPC as a broadcast approach for two-way relay configurations has been reported in several studies for different contexts (e.g. Chen and Yener, 2010; Chen et al., 2010; Oechtering and Boche, 2008); in addition a combination of SPC with network coding appropriate for asymmetric two-way relay topologies is discussed in Park and Oh (2009).

Although there is a lot of work on the design of efficient two-way relay protocols, the majority of them assume perfect channel estimation which is not always a realistic assumption. The impact of an imperfect channel estimation on the achievable system performance is a classical problem in the literature. In Medard (2000) the authors characterized the Shannon capacity of a conventional single-input single-output (SISO) network under imperfect channel estimation. A related power allocation strategy that maximizes the achieved capacity of SISO with imperfect

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channel estimation has been proposed in Klein and Gallager (2001). The impact of an imperfect channel estimation on the capacity performance of a MIMO network as well as a related waterfilling power allocation (WF-PA) strategy have been proposed in Yoo and Goldsmith (2006). On the other hand, the impact of channel estimation error on the reception reliability of a decode-and-forward two-way relay channel with physical layer network coding has been reported in Ding and Leung (to appear) in terms of error probability. In addition, different channel estimation techniques for a two-way relay channel are discussed in Jiang et al. (2010) (and references therein), while the achieved sum-rate for a two-way relay channel with AF and imperfect channel estimation is investigated in Panah and Heath (2010), Jia and Vosoughi (2011). However, the impact of an imperfect channel estimation on the achievable performance (rate) of a MIMO two-way relay protocol with SPC from an information theoretic standpoint is still an open problem in the literature.

In this paper we study the effects of imperfect channel estimation on the achievable rate region of a MIMO MABC-SPC two-way relay protocol. By extending the work presented in Hammerstrom et al. (2007) and Yoo and Goldsmith (2006), we characterize the information theoretic performance of the MIMO MABC-SPC protocol under imperfect channel estimation for two main CSI assumptions: (a) without CSI at the users where a symmetric power allocation between the two data flows is used (b) with an imperfect CSI at both users where a WF-PA is employed. Another issue that is discussed through the paper is the impact of the SPC power split on the achievable system performance. We show that an appropriate power split between the two data flows can maximize the achievable performance by simultaneously supporting user fairness. In addition a power split that maximizes the achievable sum-rate is investigated. A theoretical framework that calculates the optimal SPC power split between the two data flows for both optimization targets (fairness, sum-rate) is proposed. We show that the optimal SPC allocation is common for both CSI assumptions and independent of the instantaneous channels; a result that makes the proposed scheme suitable for applications with critical complexity constraints. To the best of our knowledge the analysis of a MIMO MABC-SPC under an imperfect channel estimation as well as the related SPC power split scheme are reported for first time in this paper.

The rest of this paper is organized as follows. In Section 2 we present the system model and we describe the considered MIMO MABC-SPC protocol as well as its related achievable rate region. In Section 3 we present the two CSI assumptions and we introduce a SPC power split under a user fairness constraint as well as for a sum-rate maximization. Numerical results are shown and discussed in Section 4, followed by concluding remarks in Section 5.

Notation: Upper case and lower case bold symbols denote matrices and vectors, respectively. $\mathrm{Tr}(\mathbf{X})$ denotes the trace of a matrix $\mathbf{X}$, $\mathbf{I}_n$ denotes the identity matrix of order n, $\log(\cdot)$ denotes the logarithm of base 2, $\mathbb{E}[\cdot]$ represents the expectation operator and the superscript H denotes the Hermitian transpose operation.

2. System model

We assume a three-node MIMO two-way relay channel consisting of two users A and B and a shared relay node R. All nodes are equipped with $M>1$ antennas and both users can establish communication using a MABC-SPC cooperative protocol. Figure 1 depicts the system model and the two phases of the cooperative protocol. We consider flat fading spatially uncorrelated Rayleigh MIMO channels where $\mathbf{H}_{Ri}\in\mathcal{C}^{M\times M}$ with $i\in\{A,B\}$ denotes the channel matrix for the $i\rightarrow R$ link. The entries of the channel matrices $\mathbf{H}_{Ri}$ are independently and identically distributed (i.i.d.) zero-mean circularly symmetric complex Gaussian (ZMCSCG) random variables with unit variance (i.e. $\mathrm{rank}(\mathbf{H}_{Ri})=M$). In addition, the channel matrices remain constant for the whole transmission (during the two phases of the adopted cooperative protocol) and change to an independent realization for the next transmission. We assume that the channel matrices are subject to a channel estimation error and therefore are *imperfectly* known at the receivers with a MMSE estimation error $\mathbf{E}_i\triangleq\mathbf{H}_{Ri}-\widehat{\mathbf{H}}_{Ri}$ for the $R\rightarrow i$ link, where the entries of $\mathbf{E}_i$ are ZMCSCG with variance $\sigma_{\epsilon_i}^2$ and the entries of $\widehat{\mathbf{H}}_{Ri}$ are also i.i.d. ZMCSCG with variance $1-\sigma_{\epsilon_i}^2$ (Yoo and Goldsmith, 2006). The two phases of the MABC-SPC cooperative protocol are described as follows:

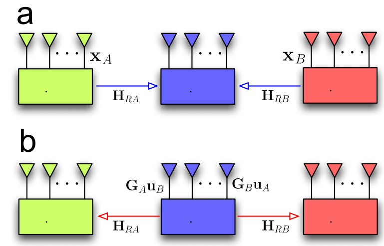

Fig. 1. The system model: (a) the first phase of the protocol, (b) the second phase of the protocol.

*Phase*1: In the first phase of the protocol both users transmit their messages to the common relay by forming a conventional MIMO multiple-access channel. At the relay node the received signal can be expressed as

$$\mathbf{r}_R=\mathbf{H}_{RA}\mathbf{x}_A+\mathbf{H}_{RB}\mathbf{x}_B+\mathbf{n}_R, \tag{1}$$

where $\mathbf{x}_i\in\mathcal{C}^{M\times 1}$ denotes the transmitted message for the i-th user, $\mathbf{n}_R\sim\mathcal{NC}(0,\sigma_n^2\mathbf{I}_M)$ represents a noise vector having ZMCSCG entries of variances σ_n^2 and both users transmit subject to a power constraint $\mathrm{Tr}(\mathbf{P}_i)\le P$ where $\mathbf{P}_i\triangleq\mathbb{E}[\mathbf{x}_i\mathbf{x}_i^H]$ denotes the input covariance matrix. Due the considered channel estimation error at the relay node, the instantaneous rate region is the closure of the convex hull of the set points $(\mathcal{R}_{A_1},\mathcal{R}_{B_1})$ satisfying (Tse, 2005, Section 10.1.2):

$$\mathcal{R}_{A_1}\le I_{A_1}\triangleq\log\det\left(\mathbf{I}_M+\frac{\widehat{\mathbf{H}}_{RA}\widehat{\mathbf{H}}_{RA}^H}{\sigma_n^2+(\sigma_{\epsilon_A}^2+\sigma_{\epsilon_B}^2)P}\right), \tag{2}$$

$$\mathcal{R}_{B_1}\le I_{B_1}\triangleq\log\det\left(\mathbf{I}_M+\frac{\widehat{\mathbf{H}}_{RB}\widehat{\mathbf{H}}_{RB}^H}{\sigma_n^2+(\sigma_{\epsilon_A}^2+\sigma_{\epsilon_B}^2)P}\right), \tag{3}$$

$$\mathcal{R}_{A_1}+\mathcal{R}_{B_1}\le I_{\mathrm{sum}}\triangleq\log\det\left(\mathbf{I}_M+\frac{\widehat{\mathbf{H}}_{RA}\widehat{\mathbf{H}}_{RA}^H+\widehat{\mathbf{H}}_{RB}\widehat{\mathbf{H}}_{RB}^H}{\sigma_n^2+(\sigma_{\epsilon_A}^2+\sigma_{\epsilon_B}^2)P}\right), \tag{4}$$

where the above expressions consist of a generalization of the analysis presented in Yoo and Goldsmith (2006, Eq. (7)).

*Phase*2: In the second phase of the protocol, the relay node re-encodes the users' messages using the same or a different codebook with the first phase of the protocol and broadcasts them to both users via a SPC scheme (Tse, 2005, Section 6.2.2; Cover and Thomas, 2006, Section 15.6). With SPC, the relay node superposes the sources' messages in the modulation domain (Larsson and Vojcic, 2005) (e.g. linear combination of the two signals) with an appropriate power split and broadcasts the resulting signal without further processing. It is worth noting that SPC is an efficient approach in order to boost the broadcast performance and achieve capacity.