Available online at www.sciencedirect.com
ScienceDirect
Geoderma 140 (2007) 346–352
GEODERMA
www.elsevier.com/locate/geoderma

Spatially-Balanced Complete Block designs for field experiments

H.M. van Es[a,*], C.P. Gomes[b,c], M. Sellmann[b], C.L. van Es[c]

[a] *Department of Crop and Soil Sciences, Cornell University, Ithaca, NY 14882, United States*
[b] *Department of Computing and Information Sciences, Cornell University, Ithaca, NY 14882, United States*
[c] *Department of Applied Economics and Management, Cornell University, Ithaca, NY 14882, United States*

Abstract

Spatial heterogeneity in fields may affect the outcome of experiments. The conventional randomized allocation of treatments to plots may cause bias and variable precision in the presence of trends (including periodicity) and spatial autocorrelation. Agricultural scientists appear to mostly use conventional experimental designs that are susceptible to adverse affects from field variability. The objectives of this research were to (i) quantify the use of different experimental designs in agronomic field experiments, and (ii) develop spatially-balanced designs that are insensitive to the effects of both trends and spatial autocorrelation. A review was performed of all research efforts reported in Volumes 93–95 of the Agronomy Journal and the frequency of various experimental designs was determined. It showed that the vast majority (96.7%) of agronomic field experiments are implemented through Randomized Complete Block (RCB) designs. The method of simulated annealing was used to develop Spatially-Balanced Complete Block (SBCB) designs based on two objective functions: promoting spatial balance among treatment contrasts, and disallowing treatments to occur in the same position in different blocks, when possible. SBCB designs were successfully developed for designs up to 15 treatments and 15 replications. Square SBCB designs were realized as Latin Squares, and perfect spatial balance was obtained when feasible. SBCB designs are simple to implement, are analyzed through conventional ANOVAs, and provide protection against the adverse effects of spatial heterogeneity, while randomized allocation of treatments still ensures against user bias.

Keywords: Experimental design; Geostatistics; Autocorrelation; Trend; Periodicity; Agronomy

Field experiments in agronomy and related disciplines have traditionally been affected by soil heterogeneity. This is especially of concern when treatment effects are small and soil variability is high, as this inflates the error term. Intrinsic soil variability is the result of the geological, hydrological, and biological factors that affect pedogenesis. The fact that soils are routinely mapped suggests that areas can be identified that are relatively uniform, but more recent research suggests that soils generally constitute a continuum with variability at different scales (van Es, 2002).

The structure of soil variability has important implications for the design of experiments. Most agronomic field experiments are based on the concepts of replication, local control (blocking) and randomization (Atkinson and Bailey, 2001). Replication allows for estimation of the experimental error by applying treatments to different plots under the same experimental conditions. Sufficient replication is needed to distinguish treatment effects from background variability. Blocking is used in field experiments to control the adverse effects of soil heterogeneity. Yates (1936) extended this concept by proposing incomplete blocks where the smaller units are assumed to adhere better to the assumption of uniformity.

The use of randomization has been justified in many ways. Its basic purpose is to remove bias from the estimation of treatment effects (Atkinson and Bailey, 2001), and to equalize the error over all treatment differences (Yates, 1939; Fagroud and van Meirvenne, 2002). Randomization is often considered the best protection and assurance against malicious manipulation of plot layout. Randomization is also believed to better justify the assumption of normal errors. A concern with randomization is the possibility of undesirable outcomes such as treatments being repeatedly located in the same location in different blocks, and treatment pairs being repeatedly located in adjacent positions. This poses no concern when variability is truly random and stationary, but agricultural scientists often admit to minor adjustments to randomized designs when treatment allocations appear undesirable.

---

* Corresponding author.
E-mail address: hmv1@cornell.edu (H.M. van Es).

0016-7061/$ - see front matter © 2007 Elsevier B.V. All rights reserved.
doi:10.1016/j.geoderma.2007.04.017

1. Accounting for nonstationarity

The common assumption in experimental design is that observations y_i are realizations of a random variable Y_i which is independently distributed with the expectation of Y_i being constant (stationary) in the experimental domain:

$$E(Y_i) = \mu \text{ for all } i, \tag{1}$$

and the variance, σ^2, being constant and estimable:

$$E[(Y_i - \mu)]^2 = \sigma^2 \text{ for all } i \tag{2}$$

μ (mean) and σ are often assumed to be parameters of a normal (Gaussian) probability distribution function, thereby allowing for a series of powerful statistical testing procedures. Past research demonstrated that these assumptions are generally erroneous for agricultural fields, and common deviations from the above model are:

- *Nonuniformity of the mean (first-order nonstationarity)*: Within the experimental domain, the land property cannot be assumed to have the same expected value (i.e., Eq. (1) is invalid), but shows structural variation through a trend or discontinuity: The presence and significance of a simple field trend can be identified (David, 1977; Davidoff et al., 1986). A special case of first-order stationarity is the presence of periodicity or cyclical trends, which tend to be associated with cultural practices such as ridge and furrow patterns, wheel traffic, etc., and may be detected by spectral analysis (McBratney and Webster, 1981).
- *Spatial autocorrelation*: This implies that the assumption of independence among observations is incorrect (Nielsen et al., 1973; Vieira et al., 1981; Russo and Bresler, 1981). In such cases, Y_i is considered to be a *regionalized* variable and the variance is expressed in terms of the relative spatial location (h):

  $$E(Y_i - Y_{i+h})^2 = 2\gamma_\iota(h) \text{ for all } i \tag{3}$$

  or

  $$E[(Y_i - \mu_\iota)(Y_{i+h} - \mu_\iota)] = C_i(h) \text{ for all } i \tag{4}$$

  where $\gamma_\iota(h)$ and $C_i(h)$ are the semivariogram and autocovariance function, respectively, which can be estimated to verify the presence of autocorrelation. The use of blocking is an implicit recognition of the common presence of spatial autocorrelation and the fact that variance generally increases with scale, i.e., smaller experimental areas have lower variability than larger ones.

Student (1938), as also cited by Atkinson and Bailey, 2001) recognized that field trends can affect the outcome of experiments and argued that plot allocations are "balanced" rather than randomized to reduce bias and the variance of the estimators of treatment differences. Jeffreys (1939) concluded that 'one should balance or eliminate the larger systematic effects first, and then randomize the rest', as is done in randomized block designs. Standard analyses (ANOVA) generally are considered to yield valid estimates of treatment effects in the presence of trends and spatial autocorrelation (Brownie and Gumperts, 1997), but detrending methods (Kirk et al., 1980; Tamura et al., 1988) and nearest neighbor analysis and related techniques (e.g., Papadakis, 1937; Wilkinson et al., 1983; Gill and Sukla, 1985) have been successfully employed to improve the precision of estimators of treatment effects.

2. Spatial autocorrelation and design

van Es and van Es (1993) evaluated the spatial nature of randomized arrangement of plots in RCB designs, and determined its effect on the outcome of experiments. Under the common condition of spatial autocorrelation, the distance between plots affects the error variance, efficiency and the outcome of tests (Martin, 1986). If the distance between plots (h_p) equals unity when they are adjacent, the mean distance ($\mu_{h\mathrm{p}}$) associated with any two treatment contrasts increases with the number of treatments (t) in an experiment (van Es and van Es, 1993):

$$\mu_{h\mathrm{p}} = (t+1)/3 \tag{5}$$

This implies that experiments with larger numbers of treatments in (complete) blocks have higher experimental errors, assuming spatial autocorrelation, than those involving lower number of treatments. Also, the spatial nature of randomization is such that the mean distance for any two treatment contrasts has higher variance ($\sigma^2_{h\mathrm{p}}$) with increasing number of treatments, but decreases with the number of replications, r (van Es and van Es, 1993):

$$\sigma^2_{h\mathrm{p}} = (t-2)(t+1)/18r \tag{6}$$

This implies that, when randomized plot allocation is used within blocks, high discrepancy will exist in the spatial distance associated with treatment contrasts when the blocks are large and the number of replications low. It was concluded from probability distributions and a simulation study involving wheat yield uniformity trial data that commonly-used randomization and replication in RCB designs may result in unequal precision in treatment comparisons and erroneous assumptions about test confidence levels in the presence of spatial autocorrelation. Similarly, it can be argued that the presence of field trends or periodicity may generate false treatment effects under certain randomization realizations if some treatments are disproportionally represented in areas of high or low fertility. Incomplete block designs provide some protection against spatial imbalance and improve efficiency (van Es et al., 1989; Lopez and Arrue, 1995; Watson, 2000). Others (e.g., Cheng and Steinberg, 1991; Watson, 2000; Fagroud and van Meirvenne, 2002; Martin et al., 2004) have addressed this concern by considering spatial autocorrelation or trend structures, in some cases from prior soil or crop information, to optimize field designs. Concerns with such approaches are that the design process becomes more costly and cumbersome, and that the autocorrelation structure is