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Probability fields revisited in the context of ensemble Kalman filtering

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SUMMARY

Hu et al. (2013) proposed an approach to update complex geological facies models generated by multiple-point geostatistical simulation while keeping geological and statistical consistency. Their approach is based on mapping the facies realization onto the spatially uncorrelated uniform random numbers used by the sequential multiple-point simulation to generate the facies realization itself. The ensemble Kalman filter was then used to update the uniform random number realizations, which were then used to generate a new facies realization by multiple-point simulation. This approach has not a good performance that we attribute to the fact that, being the probabilities random and spatially uncorrelated, their correlation with the state variable (piezometric heads) is very weak, and the Kalman gain is always small. The approach is reminiscent of the probability field simulation, which also maps the conductivity realizations onto a field of uniform random numbers; although the mapping now is done using the local conditional distribution functions built based on a prior statistical model and the conditioning data. Contrary to Hu et al. (2013) approach, this field of uniform random numbers, termed a probability field, displays spatial patterns related to the conductivity spatial patterns, and, therefore, the correlation between probabilities and state variable is as strong as the correlation between conductivities and state variable could be. Similarly to Hu et al. (2013), we propose to use the ensemble Kalman filter to update the probability fields, and show that the existence of this correlation between probability values and state variables provides better results.

1. Introduction

The ensemble Kalman filter (EnKF) Evensen (1994, 2003), is an effective and computationally efficient data assimilation method, which has received much attention in the inverse modeling community, since it can be applied for the inversion of the parameters controlling a non-linear state-transfer function given some state observational data. The EnKF is optimal for the case in which the state-transfer function is linear and parameters and state variables are multiGaussian (Aanonsen et al., 2009); it has proven to work remarkably well for non-linear state-transfer functions; but it has failed when trying to deal with non-Gaussian fields (Simon and Bertino, 2009; Chen et al., 2009; Sun et al., 2009). Recently, several methods have been developed trying to handle non-Gaussianities in EnKF; for example, those combining the EnKF and a Gaussian mixture model (Franssen and Kinzelbach, 2008; Gu and Oliver, 2007), those using iterative EnKF (Franssen and Kinzelbach, 2008; Gu and Oliver, 2007), those combining the EnKF with Gaussian anamorphosis (also referred as normal-score transform) (Bertino et al., 2003; Béal et al., 2010; Zhou et al., 2011; Xu et al., 2013b) and those combining the EnKF with multiple-point geostatistics (Hu et al., 2013).

Considering that the strength of multiple-point geostatistics is dealing with non-Gaussian fields (Guardiano and Srivastava, 1993; Strébelle, 2000), Hu et al. (2013) proposed a method to use the EnKF with non-Gaussian reservoir models by mapping facies onto the uniform random numbers used to generate them. In multiple-point geostatistical simulation (MPS), realizations are generated using the sequential simulation principle (Gómez-Hernández and Journel, 1993), whereby each node of the grid is visited, a local conditional distribution is constructed, and then a uniform random number is generated that is used to draw a value from the conditional distribution. There is a unique relationship between the (independent) uniform random numbers and the attribute values; therefore, one can envision using the random numbers as the parameters to be updated by the EnKF algorithm, and thus preserving the non-Gaussian features that are built into the calculation of the conditional distributions. The idea is very clever because once you fix all other parameters in the MPS algorithm, that is, training image, size of search neighborhood to look for conditioning data, maximum number of conditioning data to retain, path for the sequence in which the nodes are visited, etc.,

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0022-1694/

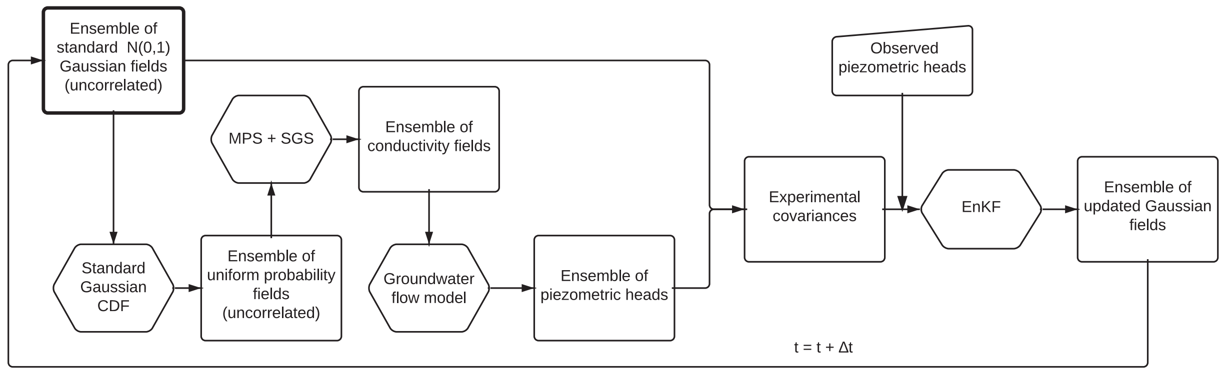

Fig. 1. Work flow for the unconditional probability field method. The starting step is highlighted in bold.

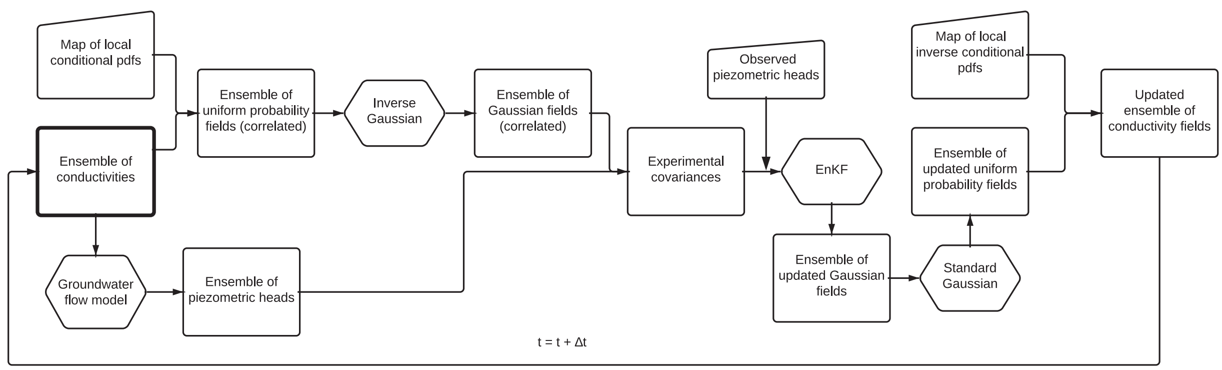

Fig. 2. Work flow for the conditional probability field method. The starting step is highlighted in bold.

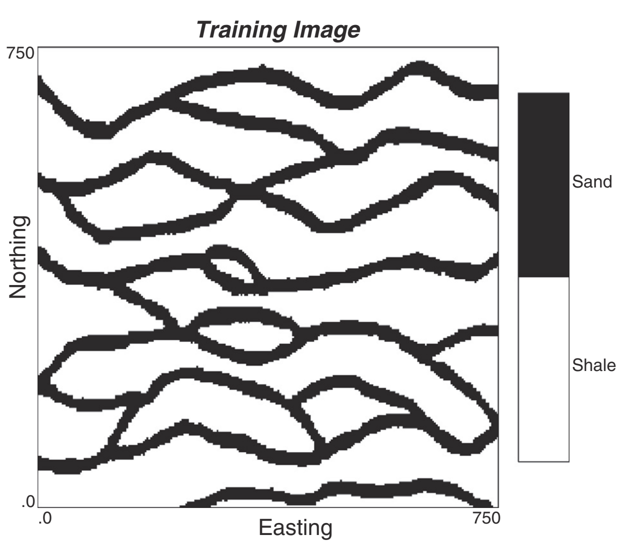

Fig. 3. Training image.

you can modify locally or globally the field of uniform random numbers to generate a new reservoir model.

The initial objective of the work by Hu et al. (2013) was to assimilate production data onto binary facies models; the mapping of the uniform probability realization onto a facies realization (a realization consisting of only two numbers) has the additional interest of finding a mapping of a discrete field onto a continuous one, since the latter will be amenable of treating by the EnKF. The method proposed by Hu et al. (2013) simply applies the standard EnKF to the uniform random numbers, instead of onto the facies values.

We have tested the method by Hu et al. (2013) in the context of assimilating piezometric heads in an aquifer and we have found that the method does not perform as well as expected, at least for the case analyzed hereafter. We think that this underperformance is due to the very weak cross-correlation that there is between the uniform numbers and the state variables. Recall that the EnKF proceeds in two steps: forecast and analysis. The forecast step presents no problem, it is based in the solution of the numerical model appropriate to the process being studied. The analysis step is the one in which the approach by Hu et al. (2013) fails. In the analysis step, discrepancy between predicted and observed states at observation locations is used to update the parameters driving the state-equation. This update is proportional to the said discrepancy, but also to what is called the Kalman gain, which is a function of the auto- and cross-covariances of parameters and state variables. When the parameters being updated are uniform random numbers that are uncorrelated in space, the auto-covariance of the parameters and the cross-covariance are very weak, resulting in a very small Kalman gain. The net effect is that during the analysis step the update of the uniform random field

Table 1
Parameters of the random functions describing the spatial continuity of the sand and shale log-conductivities.

Facies	Proportion	Mean (ln(m/d))	Std. dev. (ln(m/d))	Variogram type	λ_x (m)	λ_y (m)	Sill
Sand	0.35	2.3	1.0	Exponential	48	24	1
Shale	0.65	−3.5	0.6	Exponential	24	24	0.35