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Spatial analysis of aquifer response times for radial flow processes: Nondimensional analysis and laboratory-scale tests

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SUMMARY

A fundamental concept in groundwater hydrology is the notion of steady state, or equilibrium conditions. When a system at some initial steady state condition is perturbed by pumping, a transient cone of depression will develop and the system will approach a new steady state condition. Understanding the time scale required for the transient process to occur is of practical interest since it would help practitioners decide whether to use a steady state model or a more complicated transient model. Standard approaches to estimate the response time use simple scaling relationships which neglect spatial variations. Alternatively, others define the response time to be the amount of time taken for the difference between the transient and steady state solutions to fall below some arbitrary tolerance level. Here, we present a novel approach and use the concept of mean action time to predict aquifer response time scales in a two-dimensional radial geometry for pumping, injection and recovery processes. Our approach leads to relatively simple closed form expressions that explicitly show how the time scale depends on the hydraulic parameters and position. Furthermore, our dimensionless framework allows us to predict the response time scales for a range of applications including small scale laboratory problems and large scale field problems. Our analysis shows that the response time scales vary spatially, but are equivalent for pumping, injection and associated recovery processes. Furthermore, the time scale is independent of the pumping or injection flow rate. We test these predictions in a laboratory scale aquifer and find that our physical measurements corroborate the theoretical predictions.

1. Introduction

Population growth and associated industrial and agricultural activities can have considerable impact on groundwater resources. Since groundwater plays a significant role in our social and economic wellbeing, understanding groundwater responses to natural and anthropogenic changes is important. Several studies have examined various properties of groundwater flow processes using different tools including numerical or analytical models, field investigations and laboratory experiments (e.g. Theis, 1935; Freeze and Witherspoon, 1966; Bredehoeft et al., 1982; Hantush, 2005). Many of these studies have included radial flow problems to investigate pumping, injection and recovery processes.

A common concept used in groundwater modeling is defining a steady state (or equilibrium) flow condition. When a forcing condition on a system at equilibrium is changed, the system will undergo a transient response to approach a new equilibrium state. A point of interest is to understand the amount of time taken for the system to reach steady state. Strictly speaking, from a mathematical point of view, an infinite amount of time is required for the system to asymptote to steady state conditions. However, this strict mathematical definition is impractical because we can never wait for an infinite amount of time. Therefore, we wish to estimate a "sufficiently long period" of time that is required for the system to "effectively" reach steady state (Schwartz et al., 2010). However, the concept of a "sufficiently long period" is subtle.

A change in flow conditions at a pumping or injection well will eventually influence regions further away from the well, potentially over very large areas, including distant boundary conditions. When the flow rate at a pumping or injection well is altered, a transition pattern, often called a cone of depression, propagates through the aquifer with time. Understanding the amount of time required for a transient system to effectively relax to equilibrium can help us decide whether to use a steady state model or a more complicated transient model to describe the groundwater flow process (Simpson et al., 2013; Jazaei et al., 2014).

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Nomenclature

Notation	
B'	aquifer thickness [L]
C_1, C_2, C_3	integration constants
$F(t\|r)$	CDF for the dimensionless models [–]
$F(t'\|r')$	CDF for the dimensional models [–]
$f(t\|r)$	PDF for the dimensionless model [–]
$f(t'\|r')$	PDF for the dimensional model [1/T]
$g(r)$	$h_\infty(r) - h_0(r)$
$h(r,t)$	dimensionless hydraulic head [–]
$h'(r',t')$	dimensional hydraulic head [L]
h^*	characteristic hydraulic head [L]
$h_0(r)$	dimensionless initial hydraulic head [–]
$h'_0(r')$	dimensional initial hydraulic head [L]
$h_\infty(r)$	dimensionless steady hydraulic head [–]
$h'_\infty(r')$	dimensional steady hydraulic head [L]
$h(1,t)$	dimensionless hydraulic head at the boundary [–]
$h'(R',t')$	dimensional hydraulic head at the boundary [L]
K'	hydraulic conductivity [L/T]
L	aquifer length [L] (Gelhar and Wilson, 1974)
$M(r)$	dimensionless mean action time (MAT) [–]
m	positive integer constant [–]
n	average porosity [–] (Gelhar and Wilson, 1974)
Q'	flow rate at the well [L^3/T]
Q'_P, Q'_{I1}, Q'_{I2}	flow rate at the well in Experiment-P, Experiment-I1 and Experiment-I2, respectively [L^3/T]
r	dimensionless location from the well center [–]
r'	dimensional location from the well center [L]
r^*	characteristic length [L]
r'_w	dimensional radius of the well [L]
r_1, r_2	dimensionless radial distance of monitoring points from the well [–]
r'_1, r'_2	dimensional radial distance of monitoring points from the well [L]
R'	dimensional location of the boundary from the well center [L]
S	aquifer storage coefficient [–]
T'	aquifer transmissivity [L^2/T]
t	dimensionless time [–]
t'	dimensional time [T]
t^*	characteristic time [T]
t_h	hydraulic response time [T] (Gelhar and Wilson, 1974)
$V(r)$	dimensionless variance of action time (VAT) [–]
α	r'_w/R' [–]
β	$\frac{Q'}{2\pi T' h'_0 \alpha}$ [–]
δ	parameter indicating a specific time [T]
$\tau(r)$	dimensionless time scale $M(r) + m\sqrt{V(r)}$ [–]
$\phi(r)$	$g(r)[V(r) + T(r)^2]$ [–]

The concept of aquifer response time has been analyzed previously for various groundwater problems. Theis first considered the response of a groundwater system to pumping by solving a mathematical model describing the transient flow near a pumping well in an infinite aquifer (Theis, 1935). After this initial study, Theis then considered the factors controlling the response time (Theis, 1940). These factors include the aquifer transmissivity, T; the storage coefficient, S; and the length scale of the problem. Theis concluded that the rate at which the cone of depression spreads is proportional to T and inversely proportional to S. Later, other researchers presented simpler scaling formulas to estimate the aquifer response time scale (e.g. Gelhar and Wilson, 1974; Townley, 1995; Erskine and Papaioannou, 1997; Manga, 1999; Haitjema, 2006). For example, Gelhar and Wilson (1974) suggest that the hydraulic response time is $t_h = nL^2/3T$, where n is the average porosity and L is the aquifer length. Such scaling formulas suggest a constant time scale for the entire system and do not provide any information about how the time scale depends on position. Other studies (e.g. Schwartz et al., 2010; Kooi et al., 2000; Rousseau-Gueutin et al., 2013) define the response time as the amount of time taken for the difference between the transient and steady state solutions to fall below some tolerance. For example Rousseau-Gueutin et al. (2013) define the aquifer response time to be the amount of time required for 95% of the transient head changes to have occurred. This definition does not lead to a simple closed form expression.

Recently, we presented a different framework to quantify the aquifer response time scale (Simpson et al., 2013; Jazaei et al., 2014). Our analysis provides explicit mathematical expressions showing how the response time scale depends on position, aquifer properties and boundary conditions. This approach does not require any predefined thresholds, and avoids the need for solving the transient flow problem. However, our previous analyses were limited to one-dimensional Cartesian problems in which flows where driven by a surface recharge conditions, or changes at the interface between the surface water and groundwater. In contrast, here we analyze the time scale of a two-dimensional radial system, in which the transition between different steady state conditions is driven by flow changes at the pumping or injecting well. Our analysis is relevant for both converging and diverging flows and we employ a nondimensional framework which leads to more elegant, generalized results, which can be used to explain the difference between smaller scale laboratory flow conditions and larger scale field conditions.

Our approach involves analyzing the first and second moments of the transition time distribution, which is similar to the way in which some previous studies have used temporal moment analysis to investigate spatial variations in hydraulic conductivity (Li et al., 2005; Zhu and Yeh, 2006). We note, however, these previous studies were focusing on analyzing the hydraulic conductivity fields, and did not consider using moment analysis to derive expressions for the aquifer response time scales.

The objective of the present work is to develop a framework to quantify the spatial variations in response time scales under radial flow conditions. We investigate pumping, injection and recovery processes to understand how their response time scales depend on hydraulic and geometric properties of the aquifer. We employ two mathematical concepts, known as the mean action time (MAT) and the variance of action time (VAT) in this analysis. We employ a dimensionless framework that can be used to study both large scale field problems as well as small scale laboratory problems. Our theoretical predictions are tested using new datasets from laboratory scale experiments.

2. Mathematical model

In this section we first use a dimensional radial flow model to define a simpler and more general dimensionless model. Primed variables denote dimensional quantities and unprimed variables denote dimensionless quantities.