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A MODFLOW package to linearize stream depletion analysis

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SUMMARY

The conventional numerical method is computationally intensive and prone to numerical noises for stream depletion analyses using MODFLOW. In this study, a new MODFLOW package has been developed to improve the computational efficiency and reduce the noises for each simulation. Using the assumption of unchanged flow coefficients between the baseline and scenario runs, the nonlinear groundwater flow system is linearized for solving the flow equations. The new package has been successfully applied to a regional groundwater model in Nebraska. The results show that the numerical noises, commonly identified in conventional approach, have been significantly reduced and a twenty-fold speedup has been achieved for a regional groundwater model in Nebraska. The results suggest this package can be adapted to be a component of optimization tools for water management scenario analyses especially when a large number of scenario model runs are involved.

1. Introduction

Stream depletion induced by groundwater pumping can be computed with the analytical or numerical methods (Barlow and Leake, 2012). Since 1940s (Theis, 1941), analytical solutions for stream depletion have been actively sought for understanding the groundwater flow system in confined and unconfined aquifers, with and without streambed clogging, as well as partially and fully penetrating streams (Glover and Balmer, 1954; Hantush, 1965; Hunt, 1999; Hunt and Scott, 2005; Theis, 1941). In practice, however, numerical methods are still favorable for integrated water management due to, primarily, the limitation of analytical solution in depicting the complex stream–aquifer systems (Chen and Shu, 2002; Chen and Yin, 2001; Sophocleous et al., 1995).

Conventionally, stream depletion analyses begin with the baseline model that is a calibrated model considering all the existing stresses on the stream–aquifer system. The stream depletion is simulated by imposing an additional stress such as withdrawal from a pumping well to the baseline groundwater model, as a scenario run in the second step. By computing the differences in groundwater flows and water budgets between the two model runs, we can quantify the effects of groundwater pumping on stream depletion. In this study, we use stream depletion rate (SDR) to represent stream depletion as a fraction of the incremental pumping rate (Barlow and Leake, 2012):

$$\mathrm{SDR} = 100\frac{Q_s' - Q_s}{Q_w} \tag{1}$$

where SDR is the stream depletion rate expressed in percentage; Q_s' and Q_s are the flow rates of the river or stream leakage for the scenario run and baseline run respectively; Q_w is the pumping rate of the new pumping well.

SDR has shown unique spatial and temporal dynamics (Barlow and Leake, 2012; Konikow and Leake, 2014; Merritt and Konikow, 2000). For example, a stream depletion map (or capture map) developed using SDR or SDR-accumulated volume can help in understanding the effects of pumping locations on surface water resources (Leake et al., 2010). With the pumping rate constant in time, SDR increases exponentially initially and may need as long as hundreds of years to stabilize (Konikow and Leake, 2014). To accurately account for the spatial and temporal patterns of SDR, the current practice is often computationally intensive by imposing additional pumping at each grid cell and initiating a new model

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run. The computation can be parallelized because each scenario run is independent. However, for a regional groundwater model with tens of thousands of grid cells, the computation cost could still be exceedingly high. The adjoint approach is also a more efficient alternative because it only requires one simulation to estimate SDR for all the cells (Griebling and Neupauer, 2013; Neupauer and Griebling, 2012).

Accuracy of the conventional method is subject to the numerical errors generated in both the baseline and scenario runs. Compared to the baseline run, the pattern of the numerical errors may be maintained or altered in the scenario run. The subtraction of stream leakages between the two model runs can either suppress or enhance the impact of the numerical errors on the SDR calculation. In regional groundwater models, numerical errors tend to be relatively large due to, in part, insufficient spatial and temporal discretization. Although using larger pumping rate for scenario runs can mitigate the impact of numerical noises, it can also alter the groundwater flow system by inducing dry cells that may discontinue stream–aquifer interactions. Therefore, the pumping rate used for the new well in the scenario run also needs to be carefully selected for each grid cell (Leake et al., 2010).

The objective of this study is to develop a new method that reduces the numerical noises and improves the computational efficiency for stream depletion analyses by linearizing the groundwater flow equations. A new MODFLOW package is developed based on this new method to facilitate the implementation of this method. The new package is applied to a regional groundwater model to examine its applicability for stream depletion analyses.

2. Methodology

In the classic analytical solution developed by Hunt (1999) for stream depletion, some ideal assumptions are made to solve the problem. In Hunt's solution, besides the assumptions of the ideal aquifer and horizontal flow, it assumes that (1) "drawdowns are small enough compared with the saturated aquifer thickness to allow the governing equations to be linearized"; (2) "changes in water surface elevation in the river created by pumping are small compared with changes created in the water table elevation". Based on these assumptions, SDR can be expressed as a function of time, aquifer hydraulic properties and the distance between the pumping well and the stream. According to Hunt's stream depletion function, SDA is independent of the pumping rate (Q_w). Despite that the volume derived from each source contributing to the pumping will change with Q_w, the relative volumetric proportion of each source contributing to the pumping only changes with the location of the well. In this study, we adopt these two assumptions to linearize the flow equation in MODFLOW.

In MODFLOW, there are generally two types of flow boundary conditions according to the flow's dependency on groundwater heads (McDonald and Harbaugh, 1988). The flows of head-independent boundary conditions, including wells and recharge, are specified directly as model inputs. In contrast, the flows of head-dependent boundary conditions need to be calculated after the groundwater flow equation is solved. The general equation of the head-dependent boundary conditions can be expressed as:

$$Q_b = C_b(h_b - h) \tag{2}$$

where Q_b is the flow from the boundary to the aquifer; C_b is the flow conductance; h_b is the boundary head which, for example, are the stream stage or ET extinction depth; and h is the groundwater hydraulic head.

MODFLOW employs a finite-difference method to calculate groundwater heads and flows. Converting the groundwater flow equation into finite-difference form yields (McDonald and Harbaugh, 1988):

$$\begin{aligned}&CC_{i-\frac{1}{2},j,k}h_{i-\frac{1}{2},j,k} + CR_{i,j-\frac{1}{2},k}h_{i,j-\frac{1}{2},k} + CV_{i,j,k-\frac{1}{2}}h_{i,j,k-\frac{1}{2}} + \Big(-CC_{i-\frac{1}{2},j,k} - CR_{i,j-\frac{1}{2},k}\\ &\quad - CV_{i,j,k-\frac{1}{2}} - CC_{i+\frac{1}{2},j,k} - CR_{i,j+\frac{1}{2},k} - CV_{i,j,k+\frac{1}{2}} + \mathrm{HCOF}_{i,j,k}\Big)h_{i,j,k}\\ &\quad + CC_{i+\frac{1}{2},j,k}h_{i+\frac{1}{2},j,k} + CR_{i,j+\frac{1}{2},k}h_{i,j+\frac{1}{2},k} + CV_{i,j,k+\frac{1}{2}}h_{i,j,k+\frac{1}{2}} = \mathrm{RHS}_{i,j,k}\end{aligned} \tag{3}$$

where CC, CR and CV are the conductance coefficients along the column, row and vertical directions, respectively; HCOF and RHS are coefficients related to the sources/sinks and storage terms; the subscripts i, j and k are used to designate the cell column, row and layer respectively; and 1/2 denotes the region between two grid cells. The boundary stresses are added to or subtracted from the groundwater storage through two terms HCOF and RHS (McDonald and Harbaugh, 1988):

$$\mathrm{HCOF} = -\sum_{k=1}^{n1} C_{b,k} - \frac{S_c}{\Delta t} \tag{4}$$

$$\mathrm{RHS} = -\sum_{k=1}^{n2} Q_{s,k} - \frac{S_c h^0}{\Delta t} - \sum_{k=1}^{n1} C_{b,k}h_{b,k} \tag{5}$$

where Q_s is the head-independent specific boundary flow; S_c is the storage coefficient; Δt is the time step length; $n1$ and $n2$ are the numbers of head-dependent and specific flow boundaries on a cell; and h^0 is the head at the end of the previous time step, or the initial head for the first time step of the simulation. In a scenario run, the changes of the flow system are usually caused by changing the Q_s term. For example, adding pumping wells or reducing groundwater recharge lead to a reduced Q_s value. The flow equations of the entire system can be expressed in the matrix form as:

$$AH = F \tag{6}$$

$$A'H' = F' \tag{7}$$

where A is a matrix of coefficients of heads; H is the vector of groundwater heads; F is the vector of RHS. Eq. (7) is the expression of the flow equation for the scenario run where the prime symbols indicate the scenario run.

Eqs. (6) and (7) are non-linear because A is related to the head-dependent flow coefficients and the boundary conductance according to Eqs. (3) and (4). For each solver iteration, A is reformulated based on the updated head values. S_c is independent of heads unless the cell is converted between unconfined and confined conditions. In the scenario run, the changes of A include deviations of CC, CR, CV and C_b, which can be attributed to the new pumping stress.

CC, CR, and CV depend on layer thickness in confined layers. For an unconfined layer, on the other hand, these coefficients become head dependent, resulting in altered values in the scenario run. In a regional groundwater model, however, the head changes between two runs are usually minimal compared with the aquifer thickness. Therefore, the first assumption of Hunt's solution can also be applied to MODFLOW for stream depletion analyses.

While C_b is a constant term in most head dependent boundary packages, C_b is a function of the groundwater head and the boundary head in some packages. In the Streamflow Routing package, for example, C_b is calculated as (Prudic et al., 2004):

$$C_{b,s} = K_s w_s L_s / M_s \tag{8}$$

where $C_{b,s}$ is the streambed conductance; K_s and M_s are the streambed hydraulic conductivity and thickness, respectively; w_s is the representative width of the stream; and L_s is the stream reach length. The streamflow varies in response to the head changes in a scenario run, because it is related to the gradient of the stream stage