Journal of Hydrology 497 (2013) 176–188

Contents lists available at SciVerse ScienceDirect

Journal of Hydrology

journal homepage: www.elsevier.com/locate/jhydrol

An entropy-based method for determining the flow depth distribution in natural channels

Tommaso Moramarco [a,*], Giovanni Corato [b], Florisa Melone [a], Vijay P. Singh [c,d]

[a] National Research Council, IRPI, Via Madonna Alta 126, 06128 Perugia, Italy
[b] Département Environnement et Agro-biotechnologies, Centre de Recherche Public Gabriel Lippmann, 41 Rue du Brill, 4422 Belvaux, Luxembourg
[c] Department of Biological and Agricultural Engineering, Texas A&M University, 321 Scoates Hall, 2117 TAMU College Station, TX 77843-2117, USA
[d] Department of Civil and Environmental Engineering, Texas A&M University, 321 Scoates Hall, 2117 TAMU College Station, TX 77843-2117, USA

ARTICLE INFO

Article history:
Received 2 November 2012
Received in revised form 7 March 2013
Accepted 1 June 2013
Available online 10 June 2013
This manuscript was handled by Konstantine P. Georgakakos, Editor-in-Chief, with the assistance of Ehab A. Meselhe, Associate Editor

Keywords:
Floods
Entropy
Streamflow measurements
Bathymetry
Remote sensing

SUMMARY

A methodology for determining the bathymetry of river cross-sections during floods by the sampling of surface flow velocity and existing low flow hydraulic data is developed . Similar to Chiu (1988) who proposed an entropy-based velocity distribution, the flow depth distribution in a cross-section of a natural channel is derived by entropy maximization. The depth distribution depends on one parameter, whose estimate is straightforward, and on the maximum flow depth. Applying to a velocity data set of five river gage sites, the method modeled the flow area observed during flow measurements and accurately assessed the corresponding discharge by coupling the flow depth distribution and the entropic relation between mean velocity and maximum velocity. The methodology unfolds a new perspective for flow monitoring by remote sensing, considering that the two main quantities on which the methodology is based, i.e., surface flow velocity and flow depth, might be potentially sensed by new sensors operating aboard an aircraft or satellite.

© 2013 Elsevier B.V. All rights reserved.

0. Introduction

One of the main problems in flow monitoring is the difficulty to carry out velocity measurements for high stages that are, however, fundamental to achieve a reliable rating curve at a river gage site. The difficulty is real and twofold. First, sampling velocity points in the lower portion of flow area may entail serious dangers for the operator during measurement. Second, there is a need for a rapid and simple measurement method when hydraulic conditions are changing rapidly. To address these issues, velocity sampling during floods can be achieved by monitoring the maximum flow velocity, u_{max}, which can be easily sampled for high stages, because its location occurs in the upper portion of the flow area (Moramarco et al., 2004; Fulton and Ostrowski, 2008). Additionally, this information can be an index to depth distribution in the cross-section. From the point of view of deterministic hydraulics, it is worth noting that u_{max} did not receive much attention in the past. Its importance was highlighted by Chiu (1987, 1989) who showed that the velocity distribution in a channel cross-section can be determined as a function of u_{max} and of the curvilinear coordinates in the physical space. This important insight was obtained by Chiu by establishing a bridge between the probability domain, wherein a probability distribution of the velocity was surmised by considering the random sampling of velocity in the flow area, and the physical space by hypothesizing the cumulative probability distribution function in terms of depth in the physical domain. Applying the principle of maximum entropy (Jaynes, 1957), Chiu (1989) derived a two-dimensional probability density function of velocity in terms of the curvilinear coordinates in the physical space. However, the estimation of this density function requires as many as six parameters and is not convenient from a practical point of view. For this reason, Moramarco et al. (2004) simplified Chiu's model by assuming that the velocity distribution written for the vertical where u_{max} occurs, named *y-axis*, can be applied to other verticals as well. In addition, in order to significantly reduce the sampling period during measurements, the *y-axis* velocity profile was applied to all verticals in the flow area considering the sampling of u_{max} only and assuming the behavior of the maximum velocity in the cross-sectional flow area represented by an elliptical or parabolic curve, according to the river cross-section geometry characteristics (Moramarco et al., 2011). In this manner, the two-dimensional flow velocity was modeled by using the entropy-based velocity profile

* Corresponding author. Tel.: +39 075 5014404; fax: +39 075 5014420.
E-mail addresses: t.moramarco@irpi.cnr.it (T. Moramarco), corato@lippmann.lu (G. Corato), f.melone@cnr.irpi.it (F. Melone), vsingh@tamu.edu (V.P. Singh).

0022-1694/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.jhydrol.2013.06.002

along each sampled vertical (Moramarco et al., 2004). Results in terms of mean flow velocity, u_m, were found satisfactory for river gage sites located in rivers with different geometric and hydraulic characteristics (Corato et al., 2011; Moramarco et al., 2011). Therefore, the method is of considerable interest for hydrological practice, because monitoring of the maximum velocity, which often coincides with the maximum flow velocity, nowadays, can be done by using non-contact survey technology as fixed radar sensors or hand-held radar units (Lee et al., 2002; Costa et al., 2006; Fulton and Ostrowski, 2008; Plant et al., 2009).

The method for estimating discharge, based on the entropy-based velocity profiles, is highly sensitive to the accuracy in estimating the flow cross-sectional area and the changes therein during floods. Indeed during high floods, the geometric characteristics of river sections may change for sediment transport. Therefore, from a hydrological practice point of view, another highly appealing aspect is to understand if a nexus exists between surface-water velocity distribution and water depths in the flow area. This would allow flow monitoring during high floods through the sampling of surface flow velocities from which the cross-section geometry of the river site might be determined. Lee et al. (2002) attempted to estimate water depths from recorded surface velocities by using a non-contact radar sensor. However, hydraulic quantities, such as energy slope and Manning's roughness, need to be assigned.

The objective of this paper is to exploit the principle of maximum entropy, similar to Chiu's analysis of two-dimensional modeling of flow velocity (Chiu, 1989). However, the entropy-based method is used here for estimating the probability density function of water depth as a function of the cumulative probability distribution function of the surface maximum flow velocity. Five gage sites, of which four along the Tiber River, central Italy, and one along the Alzette River, Luxembourg, are used to validate the method.

The paper is organized as follows: Section 1 provides an overview of the theoretical background of the principle of maximum entropy. Section 2 identifies the probability distribution of the water depth in a river cross-section by using the entropy theory, as applied to the velocity field in the river channel cross-section. Section 3 describes the results achieved for five river gage sites. Finally, the last section features the conclusions drawn from this investigation.

1. A short theoretical background on the entropy theory

Shannon (1948) first proposed a theory to maximize the information content. If a system is characterized by state, X, which can take on a finite number of states, X_j, with probability $p(X_j)$, the information $I(X)$ associated with X_j is given by the quantity:

$$I(X) = -\ln p(X_j) \tag{1}$$

$I(X)$ provides the quantity of information if the event $X = X_j$ occurs and is a measure of uncertainty associated with that event. Considering all the states or events, the mean of $I(X)$ can be defined as:

$$H(X) = \sum_j p(X_j) I(X_j) = -\sum_j p(X_j) \ln p(X_j) \tag{2}$$

Shannon defined entropy by function $H(X)$. It is noted that if $X = X_j$ with probability $P(X) \approx 0$, then, $H(X) \approx \infty$; whereas if $X = X_j$ is a certain event the entropy goes to zero. The probability distribution that maximizes the entropy is the distribution that produces greater information among those coherent with the basic knowledge of the phenomenon investigated. Therefore, this principle is used for statistical inference to solve for a probability distribution function (Singh, 1986, 2011), when the information available about the variable is limited to some average quantities, defined as constraints, such as mean, and variance. Specifically, given certain independent constraints, G_i, in the form:

$$G_i = \int_a^b \psi_i(X,p)dX \qquad i = 1, 2 \ldots n \tag{3}$$

where $\psi_i(X,p)$ are the specified functions, the density function $p(X)$ which maximizes entropy can be thus obtained (Singh, 1986) as:

$$\frac{\partial I(X,p)}{\partial p} + \sum_{i=1}^{n} \lambda_i \frac{\partial \psi_i(X,p)}{\partial p} = 0 \tag{4}$$

λ_i are the Lagrange multipliers.

Chiu (1987, 1989) pioneered the application of entropy theory in open channel hydraulics. He derived a two-dimensional flow velocity distribution as a function of maximum flow velocity, $u_{\max}$, and curvilinear coordinates in the physical space. Chiu (1988) also found a linear relation between u_m and $u_{\max}$ whose slope, $\Phi(M)$, can be estimated via linear regression of data sets of pairs (u_m, $u_{\max}$) sampled at gage sites (Xia, 1997; Moramarco et al., 2004). In the same way as Chiu did for the flow velocity distribution, we develop an entropy-based method to determine the flow depth distribution in a natural channel as a function of surface velocity.

2. Derivation of flow depth distribution

In open channel flow with maximum surface-water velocity $u_{\max S}$, let us assume that the flow depth monotonously increases from zero at the banks to a maximum value in the middle of channel. Let h be the flow depth at a surface-water velocity u_s from the bank. Then, the probability of the flow depth being less than or equal to h is $\frac{u_s}{u_{\max S}}$ and the cumulative probability distribution is:

$$F(h) = \frac{u_s}{u_{\max S}} \tag{5}$$

wherein u_s and $u_{\max S}$ represent the surface velocity and the maximum surface velocity, respectively.

Therefore, the density function $p(h)$ is:

$$p(h) = \frac{dF(h)}{dh} = \frac{dF(h)}{du_s}\frac{du_s}{dh} = \frac{1}{u_{\max S}}\frac{du_s}{dh} \tag{6}$$

Two constraints are considered for deriving $p(h)$:

$$\int_0^D p(h)dh = 1 \tag{7a}$$

and

$$\int_0^D hp(h)dh = H_m \tag{7b}$$

where D and H_m are the maximum and the mean flow depth in the river cross-section, respectively. Therefore, consideration of Eq. (3) and the two above constraints (Eqs. (7a) and (7b)) yields:

$$\begin{cases} \psi_1 = p(h) \\ \psi_2 = hp(h) \end{cases} \tag{8}$$

Considering Eq. (4) and Eq. (8), the density function $p(h)$ maximizing the entropy is obtained by solving:

$$\frac{\partial[-p(h)\ln p(h)]}{\partial p} + \lambda_1 \frac{\partial p(h)}{\partial p} + \lambda_2 \frac{\partial (hp(h))}{\partial p} = 0 \tag{9}$$

From Eq. (9), it follows:

$$-\ln p(h) + \lambda_1 + \lambda_2 h - 1 = 0 \tag{10}$$

Therefore, the expression of $p(h)$ obtained for depth is similar to that obtained by Chiu (1987) for the flow velocity:

Download English Version:

https://daneshyari.com/en/article/4576246

Download Persian Version:

https://daneshyari.com/article/4576246

Daneshyari.com