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Fluid flow regimes and nonlinear flow characteristics in deformable rock fractures

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SUMMARY

The presence of fracture roughness, isolated contact areas and the occurrence of nonlinear flow complicate the fracture flow process. To experimentally investigate the fluid flow regimes through deformable rock fractures, water flow tests through both mated and non-mated sandstone fractures were conducted in triaxial cell under changing confining stress from 1.0 MPa to 3.5 MPa. For the first time Forchheimer's nonlinear factor *b* describing flow in non-mated fractures under variable confining stress has been quantified. The results show that linear Darcy's law holds for water flow through mated fracture samples due to high flow resistance caused by the small aperture and high tortuosity of the flow pathway, while nonlinear flow occurs for non-mated fracture due to enlarged aperture. Regression analyses of experimental data show that both Forchheimer equation and Izbash's law provide an excellent description for this nonlinear fracture flow process. Further, the nonlinear flow data indicate that for smaller true transmissivity, the appreciable nonlinear effect occurs at lower volumetric flow rates. The experimental data of both mated and non-mated fracture flow show that the confining stress does not change the linear and nonlinear flow patterns, however, it has a significant effect on flow characteristics. For mated fracture flow, the slope of pressure gradient versus flow rate becomes steeper and the transmissivity decreases hyperbolically with increase of confining stress, while for non-mated fracture flow, the rate of increase of the nonlinear coefficient *b* used in Forchheimer equation steadily diminishes with the increase of confining stress. Based on Forchheimer equation and taking 10% of the nonlinear effect as the critical state to distinguish between linear and nonlinear flow, the critical Reynolds number was successfully estimated by using a nonlinear effect coefficient *E*. This method appears effective to determine critical Reynolds numbers for specific flow cases.

1. Introduction

Fluid flow through rock fractures commonly occurs in many rock engineering activities, such as mining, geothermal extraction, petroleum exploitation and reservoir storage. In the past, substantial efforts have been made in hydromechanical and hydrodynamical studies in this area (Tsang and Witherspoon, 1981; Chen et al., 2000; Nazridoust et al., 2006; Crandall et al., 2010; Ferer et al., 2011). However, the combined complexity, caused by fracture roughness and isolated contact asperities between two confined fracture walls, makes an accurate description of this flow process difficult. Furthermore, deviation from linear Darcy's law, which has been observed with the increase of flow velocity, cannot be neglected for successful flow prediction in certain flow situations and hence compounds the problem.

Conventionally, linear Darcy's law has been used to describe the laminar fracture flow at low velocity. Based on the assumption that Darcy's law is valid, the cubic law was derived by describing the fracture as two smooth parallel plates where the volumetric flow rate (Q) is linearly proportional to the cube of hydraulic aperture given by:

$$Q = Ce^3 \frac{dp}{dx} \tag{1}$$

where dp/dx is the pressure gradient over the flow length and C is a constant. C is equal to $\left(\frac{1}{12\mu} \cdot \frac{2\pi}{\ln(r_e/r_w)}\right)$ and $\left(\frac{w}{12\mu}\right)$ for radial and straight flow, respectively, in which r_e is the outer radius, r_w is the well radius and w is the fracture width. The cubic law was validated for laminar flow through parallel glass plates by Lomize (1951). However, the real rock fractures are rough and distinctly different from smooth parallel-plate model. Lomize (1951) further modified cubic law by introducing a correction factor f for open rough-walled rock fractures described by:

$$Q = \frac{C}{f} e^3 \frac{dp}{dx} \tag{2}$$

where $f = 1 + 6.0\left(\frac{\varepsilon}{e}\right)^{1.5}$, in which ε is the absolute height of asperities. Witherspoon et al. (1980) examined the validity of cubic flow for real rough rock joints with scattered contact areas and found

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that *f* falls in the range from 1.04 to 1.65. In the aspect of numerical computation, the cubic law has been assumed to be locally valid and implemented in numerical codes for coupled hydromechanical behaviour prediction. Oron and Berkowitz (1998) re-examined the validity of local cubic law in self-affine fractures by directly solving Navier–Stokes equations and found that, with the reduction of fracture aperture, the overall hydraulic conductivity decreased faster than that described by cubic law. Therefore, fracture roughness and isolated contact areas play an important role in flow equations.

Even though the nonlinear flow through rough rock fractures has been experimentally and numerically observed, the study of the reason triggering nonlinear flow occurrence in rough rock fractures is limited. Initially, the non-linear flow regime was attributed to turbulence. Hassanizadeh and Gray (1987) suggested that microscopic inertial forces and increased viscous forces may also trigger the nonlinear deviation. This was supported by joint work of Ruth and Ma (Ruth and Ma, 1992; Ma and Ruth, 1993). Panfilov et al. (2003) found that the nonlinear flow can be triggered by coupled viscous and inertial effect at low Reynolds number. Quadratic termed deviation can occur and is mainly triggered by pure inertial effect (singular effect of eddies formed inside of locally-varied cavities at higher Reynolds number) and partially by inertia-viscous cross effect. Further on, Panfilov and Fourar (2006) concluded that the influence of cross inertia-viscous forces disappears in non-periodic flow channel structure.

Zimmerman and Bodvarsson (1996) found that the nonlinear effects become appreciable at Reynolds number larger than 25 in a *priori* order-of-magnitude analysis over the viscous and inertial terms in Navier–Stokes equations. Oron and Berkowitz (1998) suggested taking the Reynolds number of 10 as the theoretical limit of linear flow. Experimental and computational flow data of Zimmerman et al. (2004) showed that the initial nonlinear deviation starts approximately at Reynolds number of 1.

The nonlinear flow can theoretically occur in a fracture with any aperture size, which can allow low and high flow velocity. However, at the laboratory experimental scale, the test is usually conducted by pressure control instead of flow velocity, and high flow velocity cannot occur due to small aperture size. A summary of the aperture sizes used in the reported nonlinear flow investigation through rock fractures was given in Table 1.

With regard to hydromechanical behaviour of jointed rock, it has been known that *in situ* stress has a direct influence on fracture aperture and hence flow characteristics (Barton et al., 1985; Roman et al., 2012). The early experimental study of the coupled hydromechanical behaviour of rock joints was conducted by Gangi (1978), Tsang and Witherspoon (1981) and Barton et al. (1985) at the laboratory scale. By field experiments at a scale of approximate 1 m, Cornet et al. (2003) investigated the influence of mechanical opening on flow distribution in a single natural fracture and found that the hydraulic aperture and mechanical opening becomes equivalent only when the mechanical aperture is larger than 15 μm, indicating that the influence of rough fracture surface on flow becomes negligible. With field experiments of water flow through a well-connected fracture network consisting of low permeable bed plane and high permeable fault, interdependent hydromechanical responses tends to be highly variable and hydromechanical response of a single discontinuity inside the fracture network is significantly influenced by boundary conditions (Cappa et al., 2005). However, few publications focused on the effects of *in situ* stress on the fracture flow regime, especially the nonlinear flow characteristics. For practical purposes, the effect of *in situ* stress on flow regimes cannot be neglected in the engineering fields and needs to be studied further.

The purpose of this paper is to investigate the flow regimes in rough rock fractures subject to changing stress confinement and estimate related nonlinear flow characteristics. Water flow tests through four axially-orientated fractured rock samples were conducted in the triaxial cell for both mated and non-mated cases. Non-mated fracture samples were prepared by displacing two fracture halves by 2 mm along the flow direction, representing rock joints typically experienced in the engineering fields.

2. Theories relevant to this study

At the microscopic scale, incompressible Newtonian flow is governed by the well-known Navier–Stokes equations:

$$\rho\left(\frac{\partial U}{\partial t}+U\cdot\nabla U\right)=-\nabla p+\mu\nabla^2 U+F \tag{3}$$

where ρ is the fluid density, U is the velocity vector of flow particle, ∇p is the pressure gradient and F is the body force vector. The terms $\rho\frac{\partial U}{\partial t}$ and $\rho U\cdot\nabla U$ represent force components caused by the rate of momentum change and convective acceleration, respectively. These two terms together represent force from inertial effect, while the term $\mu\nabla^2 U$ represents viscous force. Although Navier–Stokes equations describe the incompressible Newtonian flow very well, the convective term $\rho U\cdot\nabla U$ renders the equations to be nonlinear and make them very difficult to solve. For convenience of engineering design, macroscopic empirical governing equations have been proposed.

Linear Darcy's law has been used to describe the laminar fracture flow at low velocity given by:

$$-\nabla p=\frac{\mu}{k_0 A}Q \tag{4}$$

where k_0 is the permeability and A is the flow area. Even though Darcy's law was empirically proposed based on experimental observation of water flow through column packed sands, numerous efforts have been successfully made to theoretically prove the validity of Darcy's law using different mathematical techniques, such as the average approach used by Neuman (1977) and Whitaker (1986), and the continuum method used by Hassanizadeh and Gray (1980).

Two equations have been used to describe the nonlinear flow relationships in fracture and porous media without focusing on the transition from linear to nonlinear flow. Forchheimer (1901) proposed a zero-intercept quadratic equation to macroscopically characterise the nonlinear flow process described by:

$$-\nabla p=aQ+bQ^2 \tag{5}$$

where a and b are model coefficients, representing pressure drop components caused by linear and nonlinear effects, respectively, in which $a=\frac{\mu}{k_0 A}$. Eq. (5) can also be written as:

Table 1

Summary of aperture sizes used in reported literature on nonlinear fracture flow investigation.

Authors	Aperture size	Study type	Note
Zimmerman et al. (2004)	148.9	Experiment	Mean aperture in microns
Lucas et al. (2007)	2	Numerical computation	In arbitrary unit with a relative size of 1 for cavity depth
Ranjith and Darlington (2007)	–	Experiment	Not reported
Nowamooz et al. (2009)	409, 429, 441	Experiment	In microns