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A memory model of sedimentation in water reservoirs

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SUMMARY

We consider a one-dimensional model of water reservoir, where the sediment is diffusing according to the Fourier law modified with the introduction of a derivative of fractional distributed orders as memory formalism. The fractional order is equivalent to a time-varying diffusivity and the distributed orders represent a variety of memory mechanisms to model a sediment with a varied distribution of grain sizes. Using the Laplace transform (LT), we find the solution in the case when the flux is constant at the source and is arbitrarily given at the output. Then, the time-domain solution is obtained by means of a numerical Fourier transform. We apply a one-dimensional simplified model, with the diffusion governed by two parameters, to the Quarto Nuovo (Italy) reservoir, where the flux of sediment at the output is obtained from observed data. It is found that the flux increases when one of the parameters defining the diffusion model, the pseudo-diffusivity, is increasing or when the other parameter defining the diffusion, the order of fractional differentiation, is decreasing. When the latter parameter is nil, one obtains the classic diffusion with maximum flux.

1. Introduction

1.1. Foreword on water reservoirs and sediment diffusion

Due to the shortage of fresh water the artificial water reservoirs are of ever increasing importance. They are mostly contained by dams made of earth or concrete. Both types of reservoirs, however, are doomed to inefficiency because of the phenomenon of the diffusion of sediments filling the reservoirs. For instance, in the Italian Alps, the lifetime of the water reservoirs lies between 50 and 100 years depending on the sediments contained in their tributary streams, on their geometry and on the use which set serious constraints on their economic convenience.

Monitoring the deposits of sediment in the water reservoir in the first few years after the construction is generally sufficient to have an approximate estimate of its future efficiency which concern its investors and managers.

The phenomenon of sediment diffusion and deposit in the water reservoirs is extremely complex. Most important is the varied density distribution function of the sediment relative to its size and weight, but are also important the seasonal variation in river flow, the reservoir operation scheduling and the irregular and varying shape and volume of the reservoirs occurring at the same time of the deposit of the sediment, whose fate is influenced by the force of gravity and by the velocity of the water.

In recent times, we have been gratified by several studies concerning the estimate of the flux of sediment versus time and at different locations along the water reservoirs. To quote the works which may seem more pertinent, we begin with that of Zyryanov (1973) reporting on the silting of the Uch-Kurgansk hydroelectric station and on the silt control. Bodulski and Górski (2007) studied the silting of the Cedzyna water reservoir in Poland in the period 1972–2003, finding that the volume had decreased by 113,000 m^3. Rădoane and Rădoane (2005) analyzed the data of 138 reservoirs with relatively large volume affected by the phenomenon of silting and found that it is very serious for 11% of them and serious for 22%. Sharma and Dubey (2001) discussed the remote monitoring of the silting in water reservoirs for estimating the silting delivery rate. Cogollo and Villela (1988) provided means of estimating the sediment distribution in time and space inside a reservoir.

Chen et al. (1978) produced a model for the prediction of the deposit of sediment in reservoirs. The river is modeled by a single channel assuming that the one-dimensional flow phenomena are dominant, whereas a compound stream model approach is used to simulate the main river and the flood plains of the reservoir. Their jet theory is incorporated in a mathematical model by Lopez (1978) and the resulting flow field, computed with the scheme of finite differences, is used to route the sediment through the

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reservoir. The simulated bed profiles generated by the mathematical model compare well with measured data.

Some of the difficulties in modeling the diffusion could be tentatively overcome by introducing appropriate linear phenomenological equations at the price to loose some of the intuitive properties of the classic equations. On the other hand, the phenomenological equations may allow to find different points of view on the evolution of the phenomenon of diffusion and, possibly, new concepts.

There are many generalizations of the original diffusion equation for use in various fields of science. A relevant case for anisotropic media is the substitution of the scalar parameter of diffusion with a tensor. Another relevant case is the Fokker–Planck equation describing the time evolution of the probability density function of the position of the diffusing particles.

The diffusion equation has been generalized with the introduction of memory formalisms represented by fractional order derivatives (Wyss, 1986; Mainardi, 1993). In this work, concerning the diffusion of sediment in water reservoirs, the diffusion equation is generalized by introducing a distributed order fractional derivative to represent the effect of the various density distribution functions of the sizes and weights of the particles forming the sediment. The scope is then to present a mathematical model based on a memory formalism for the diffusion of sediment in a one-dimensional water reservoir. The more general model of sedimentation in the water reservoirs introduced here, when adequate data is available, would give the flux of sediment along the reservoirs and may possibly estimate the evolution of their efficiency and improve the capability of forecast of their lifetime. It may also be of help in selecting the sites of future reservoirs in connection with the estimate of sediments in the tributary streams.

1.2. The use of the mathematical memory formalism

The basic notion of memory functions is widely recognized in science in general and, in particular, in the fields of mathematical physics, engineering and biology. Numerous applications of mathematical memory formalisms to the description of physical phenomena have been published. We try here to recall some contributions being sure that some work will be unintentionally omitted.

Using fractional derivatives as memory formalisms Baleanu and Agraval (2006) studied the Hamilton formalism. Baleanu and Trujillo (2010) studied the Euler–Lagrange equations and Baleanu et al. (2009) studied the Newtonian law with memory. Körnig and Müller (1989) used a rheological model based on fractional calculus to estimate the anelastic properties of the crust of the Earth. Iaffaldano et al. (2006) and Di Giuseppe et al. (2010) modeled the flux of water through different types of sand using diffusion equations modified with the introduction of fractional derivatives and Schumer et al. (2009) modeled transport on the Earth's surface with a fractional advection diffusion equation.

Zhang et al. (2007) studied the impact of boundaries on the fractional advection–dispersion equation for solute transport in soil defining the fractional dispersive flux with the fractional derivatives. Murio and Mejia (2008) studied the generalized time inverse heat convection problems with fractional derivatives. Bagley and Torvik (2000a,b) discussed the problem of the existence of the order domain and the solution of distributed-order differential equations. Mainardi et al. (2008) generalized the partial differential equation of Gaussian diffusion by using the time-fractional derivative of distributed order between 0 and 1, in both the Riemann–Liouville and the Caputo sense.

The fractional derivative was also used in medicine: El-Shahed (2003) made a fractional calculus model of heart valve vibrations, Magin and Ovadia (2008) modeled the cardiac tissue electrode interface using fractional calculus and Freed and Diethelm (2008) applied the fractional derivatives in viscoelasticity for a non-linear finite-deformation theory of tissue.

The derivatives of fractional order are often used to model biological phenomena, as for instance the diffusion of fluids in organic and inorganic substances. For instance, Cesarone et al. (2005) and Caputo and Cametti (2008, 2009) introduced a fractional derivative in the diffusion equation to model the profile concentration of diffusing solutes inside cell membranes. The latter authors compared their model predictions with experimental results concerning the permeation of piroxicam, an anti-inflammatory drug, and of 4-cyanophenol through human skin in vivo, obtaining a good fit.

Caputo and Carcione (2011a,b) used fractional derivatives of distributed order to model fatigue criteria and wave simulation, respectively, while Caputo et al. (2011) applied fractional derivatives to the propagation of waves in biological dissipative media.

In seismology, Carcione et al. (2002) and Carcione (2009) described the anelastic behavior of general materials over wide frequency ranges by using fractional derivatives, in particular considering propagation with constant-Q characteristics.

In finance, the fractional derivative represents the effect of memory on the economic operators concerning their action in the markets. Scalas et al. (2000) developed a theory which fully takes into account the non-Markovian and non-local character of financial time series and Mainardi et al. (2000) pointed out the consistency of the results of Scalas et al. (2000).

In physics, Laskin (2000, 2002) applied the fractional derivative in quantum mechanics, particularly to the equation of Schrödinger discussing the difference with the original equation. Závada (2002) studied relativistic wave equations involving fractional derivatives, Raspini (2000) studied the Dirac equation with a fractional derivative of order 2/3, Magin et al. (2009) solved the Bloch equation, which relates a macroscopic model of magnetization to applied radiofrequency, in gradient and static magnetic fields, in order to detect and characterize neurodegenerative, malignant and ischemic diseases.

In information theory, Frederico and Torres (2008) studied the optimal control in the sense of the fractional Noether theorem. Introducing this derivative in the stress strain relation of elasticity is possible to model the phenomenon of dissipation of the elastic energy; that dissipation which renders harmless an earthquake at sufficiently large distances from the epicenter.

All the equations generalizing the Fourier equation, and used in the works previously mentioned, are called phenomenological, since they are not obtained from first principles only. The reputation of this type of equations has been confirmed for their important contribution given in various forms to the rapid developments of the superconductive materials. These phenomenological equations, when adequately verified with experimental data, represent a step forward with respect to the usual empirical equations which are still very useful in many branches of applied science and technology.

The fractional calculus is used here to describe a one-dimensional model of water reservoir, where the sediment is diffusing according to the Fourier law. The model is applied to the Quarto Nuovo reservoir. A list of symbols is given in Appendix A.

2. The Quarto Nuovo reservoir

The ITCOLD is the Italian research group supervising the management of the water reservoirs and of the operations of removal of their sediment according to an Italian law issued in 2006. This group studied the Quarto Nuovo reservoir which was built between 1923 and 1925 in the Italian province of Forli, along the State Road 71 from Cesena to Bagno di Romagna, at the elevation