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1D finite volume model of unsteady flow over mobile bed

Shiyan Zhang, Jennifer G. Duan *

Department of Civil Engineering and Engineering Mechanics, University of Arizona, Tucson, Arizona, USA

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SUMMARY

A one dimensional (1D) finite volume method (FVM) model was developed for simulating unsteady flow, such as dam break flow, and flood routing over mobile alluvium. The governing equation is the modified 1D shallow water equation and the Exner equation that take both bed load and suspended load transport into account. The non-equilibrium sediment transport algorithm was adopted in the model, and the van Rijn method was employed to calculate the bed-load transport rate and the concentration of suspended sediment at the reference level. Flux terms in the governing equations were discretised using the upwind flux scheme, Harten et al. (1983) (HLL) and HLLC schemes, Roe's scheme and the Weighted Average Flux (WAF) schemes with the Double Minmod and Minmod flux limiters. The model was tested under a fixed bed condition to evaluate the performance of several different numerical schemes and then applied to an experimental case of dam break flow over a mobile bed and a flood event in the Rillito River, Tucson, Arizona. For dam break flow over movable bed, all tested schemes were proved to be capable of reasonably simulating water surface profiles, but failed to accurately capture the hydraulic jump. The WAF schemes produced slight spurious oscillations at the water surface and bed profiles and over-estimated the scour depth. When applying the model to the Rillito River, the simulated results generally agreed well with the field measurements of flow discharges and bed elevation changes. Modeling results of bed elevation changes were sensitive to the suspended load recovery coefficient and the bed load adaptation length, which require further theoretical and experimental investigations.

1. Introduction

Numerical simulation of unsteady flow over mobile bed (e.g. flash flood, dam break flow) is theoretically challenging and practically important. Flash floods from extreme precipitation events in ephemeral streams are the major cause of sediment transport and channel morphologic changes (Coppus and Imeson, 2002). For example, Polyakov et al. (2010) found that 10% of rainfall events with largest sediment yield produced over 50% of the total sediment yield during a period of 34 years in eight watersheds in southern Arizona. Dam failure can also produce a flash flood well beyond the natural flow regime, which can cause intense sediment transport and significant geomorphic changes to the channel (Brooks and Lawrence, 1999). Numerical model is a useful tool for understanding and predicting the characteristics of transient flow processes. However, previous models simulating sediment transport in unsteady flows either calculated the long-term channel deposition or erosion using quasi-unsteady or unsteady approaches (Lyn, 1987; Hardy et al., 2000; Deletic, 2001; Duan and Nanda, 2006; Chen and Duan, 2008) or limited their applications to the laboratory experiments (Crotogino and Holz, 1984; Park and Jain, 1987; Bhallamudi and Chaudhry, 1991; Minh-Duc and Rodi, 2008).

Few researchers have reported the simulations of flow with discontinuity or large spatial gradient over mobile bed (Capart and Young, 1998; Fraccarollo and Capart, 2002; Cao et al., 2004; Wu and Wang, 2007). Capart and Young (1998) applied the upwind scheme in 1D numerical model and their results matched well with the experimentally observed water surface profiles and bed elevations, which showed the applicability of upwind scheme in simulating dam break flow. Fraccarollo and Capart (2002) used the approximate Riemann solver for the unsteady shallow water equation. However, their model over simplified the sediment laden flow by assuming an upper pure water layer, an intermediate water–sediment mixture layer of a constant sediment concentration, and a lower solid-like motionless layer, which neglected the interaction between suspended load and bed load. Cao et al. (2004) calculated a dam break flow over an erodible bed using Weighted Average Flux (WAF) approximate Riemann solver and SUPERBEE flux limiter to achieve the second order accuracy in space, but their study only considered the transport of suspended load. Wu and Wang (2007) have taken both suspended load and bed load into account in the simulation, and developed 1D finite volume model with an explicit Godunov-type upwind flux scheme; however,

* Corresponding author. Tel.: +1 520 626 5946; fax: +1 520 621 2550.
E-mail addresses: zhangshy@email.arizona.edu (S. Zhang), gduan@email.arizona.edu (J.G. Duan).

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their model was only the first order accurate in space. Numerical schemes used in these models include the upwind flux scheme, (Ying et al., 2004; Wu and Wang, 2007) HLL and HLLC schemes (Harten et al., 1983; Toro et al., 1994), Roe's scheme (Roe, 1981; Garcia-Navarro and Vazquez-Cendon, 2000), and WAF scheme (Toro, 1992). Except the WAF scheme is of 2nd order accuracy, the rest are 1st order accurate. Although higher order schemes often yield more accurate results, their results often have spurious oscillations so that a total variation diminishing (TVD) flux limiter needs to be employed to suppress such oscillations. Sanders and Bradford (2006) tested several limiter functions by simulating solute transport and found these limiter functions performed differently in controlling numerical dissipation. Up to now, the performances of these numerical methods have not been evaluated in simulating flow and sediment transport over mobile bed.

In addition to numerical schemes, sediment transport in unsteady flows is non-equilibrium, which means sediment transport lags instantaneous flow field due to the inability of sediment motion to immediately response to changes in flow (Phillips and Sutherland, 1989). The suspended load recovery coefficient and the bed load adaptation length are commonly used to account for the non-equilibrium transport of suspended load and bed load, respectively (Wu and Wang, 2007). Although many researchers proposed formulas to calculate the non-equilibrium adaptation length and the recovery coefficient for their numerical models (Armanini and Disilvio, 1988; Celik and Rodi, 1988; Rahuel et al., 1989; Holly and Rahuel, 1990a,b; Zhou and Lin, 1998; Belleudy, 2000; Chang and Yen, 2002; Cao et al., 2004; Duan and Nanda, 2006; Wu and Wang, 2007), no consensus has been reached among researchers. This study estimated the suspended sediment recovery coefficient by Duan and Nanda (2006) and the bed load adaptation length by Rahuel et al. (1989), which have been approved valid for non-equilibrium sediment transport simulation. Since the current research is focused on the performance of different numerical schemes, discussions of different methods in simulating non-equilibrium sediment transport are not included in this manuscript.

This paper aims to examine the capability of various numerical schemes in finite volume model for simulating the unsteady flow over mobile bed due to sediment transport. The study has applied 1st order upwind scheme, HLL and HLLC scheme, Roe's scheme and 2nd order WAF schemes for the spatial derivatives in 1D FVM model. The model is used to simulate laboratory experiments of unsteady flows over fixed and erodible beds and then a flash flood event in the Rillito River, Tucson. The performances of these schemes are evaluated by comparing the results with the laboratory and field measurements. The lag effect of sediment transport in unsteady flow is accounted by implementing non-equilibrium sediment transport models for both suspended load and bed load.

2. Mathematical model

2.1. Governing equations

The governing equation includes the St. Venant equation modified by treating the density of sediment laden flow as a variable (Eqs. (1) and (2)) as well as the sediment mass conservation equation (Eq. (3)) and the Exner equation (Eq. (4)). Eq. (5) is to solve the non-equilibrium sediment transport rate assuming that the bed load transport rate reaches equilibrium after a distance, named as the recovery length, while the non-equilibrium suspended load is accounted by employing an empirical recovery coefficient (Duan and Nanda, 2006). Fig. 1 showed a typical cross section with movable bed in which an active entrainment and deposition occurs at the surface of movable bed layer. The mathematical equations are listed below similar to those in Wu and Wang (2007):

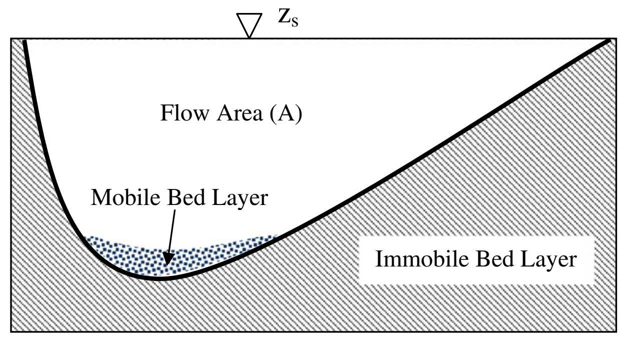

Fig. 1. Schematic drawing of a cross section with a mobile bed layer.

$$\frac{\partial(\rho A)}{\partial t} + \frac{\partial(\rho Q)}{\partial x} + \frac{\partial(\rho_b A_b)}{\partial t} = 0 \tag{1}$$

$$\frac{\partial}{\partial t}(\rho Q) + \frac{\partial}{\partial x}\left(\frac{\rho Q^2}{A}\right) + \rho g A \frac{\partial z_s}{\partial x} + \frac{1}{2} g A h_p \frac{\partial \rho}{\partial x} + \rho g \frac{n^2 Q|Q|}{A R^{4/3}} = 0 \tag{2}$$

$$\frac{\partial}{\partial t}(AC) + \frac{\partial}{\partial x}(QC) = B(E - D) \tag{3}$$

$$\frac{\partial}{\partial t}\left(\frac{Q_b}{u_b}\right) + \frac{\partial Q_b}{\partial x} = \frac{1}{L}(Q_{b*} - Q_b) \tag{4}$$

$$(1 - p_m)\frac{\partial A_b}{\partial t} \approx B(D - E) + \frac{1}{L}(Q_b - Q_{b*}) \tag{5}$$

where t = time; x = longitudinal coordinate; A = flow area; Q = flow discharge; A_b = mobile bed area; g = gravitational acceleration; z_s = water surface elevation; n = Manning's roughness; R = hydraulic radius; C = concentration of suspended load; B = width of the cross section; E = entrainment rate at the interface between bed load and suspended load; D = deposition rate at the interface between bed load and suspended load; L = non-equilibrium adaptation length; Q_{b*} = bed load transport capacity under equilibrium state; Q_b = actual bed load transport rate; ρ = density of the water–sediment mixture, where $\rho = \rho_w(1 - C_t) + \rho_s C_t$, where ρ_w and ρ_s are the density of the water and sediment, respectively, and C_t the volumetric concentration of total load sediment, calculated as $C_t = C + \frac{Q_b}{Q}$; ρ_b = density of mobile bed layer, calculated as $\rho_b = \rho_w p_m + \rho_s(1 - p_m)$, with p_m being the porosity of bed load sediment; h_p = averaged flow depth in a cross section, equivalent to the local flow depth in a rectangular flume.

2.2. Model closure

To close the sediment model, an empirical sediment transport formula for calculating the equilibrium sediment transport rate is required. This study assumes the van Rijn's equation (1984a,b) is valid for approximating the equilibrium sediment transport rate; though the method may require further investigations into its applicability to rapidly varied unsteady flows, such as dam break flow. More details about the van Rijn's approach can be found in Sturm (2001). Since the van Rijn's approach did not consider the effect of bed slope on sediment transport rate, Wu (2004) suggested to modify the effective shear stress based on the bed slope as follows:

$$\tau_* = \tau'_* + \tau_{*c}\lambda \sin\varphi / \sin\phi \tag{6}$$

$$\lambda = \begin{cases} 1 & \text{if } \varphi \leqslant 0 \\ 1 + 0.22\left(\frac{\tau'_*}{\tau_{*c}}\right)^{0.15} \exp(2\sin\varphi/\sin\phi) & \text{if } \varphi > 0 \end{cases} \tag{7}$$