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Temporal uncertainty estimation of discharges from rating curves using a variographic analysis

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SUMMARY

A rating curve provides an estimation of river discharges based on stage (water level). This estimation contains a level of uncertainty. Initial uncertainty occurs at the time of establishment of the rating curve. This may be due, for example, to the randomness of natural processes or to the inaccurate measurement of the stage. Temporal uncertainty is related to the well-known processes of erosion and deposition that modify the geometry of the river bed and, consequently, the relationship between the stage and discharge. As time goes by, temporal uncertainty of the estimated discharge from a rating curve increases. Due to the widespread use of rating curves by scientists and water resource managers, it is important to assess these related uncertainties. Several studies have taken into account initial uncertainties but none, to our knowledge, has considered temporal uncertainties. The aim of this paper is to develop a methodology to estimate the temporal uncertainty of the discharge that is estimated by the rating curve. The proposed approach is based on a variographic analysis. At the beginning of rating curve validity period, the estimated discharge is believed to be distributed as a normal distribution centered on the rating curve's estimation. The initial variance of the normal distribution, according to the initial uncertainty, is fixed so that the relative uncertainty is less than 5%. A temporal variance term, estimated using a variographic analysis, is then added to the initial variance to take into account the temporal uncertainty. This term corresponds to the mean of semi-variance between estimations separated by a given time. The proposed method has been applied to 1803 gaugings from 19 hydrometric stations located in the French Alps. The 95% confidence intervals cover 90% of 1803 gaugings. This result shows that the confidence intervals are too short. However, this may be due to an underestimation of the initial variance. The method is efficient and robust since it can be adapted to various station characteristics, such as trends in discharge series or stability of the river bed.

1. Introduction

In the fields of hydrology and hydraulics, knowledge of streamflow and its associated uncertainties are essential, both for research and engineering applications, such as water resources management, flood forecasting or spillway design. Usually, streamflow is not attained using direct measurements because this is time consuming, expensive and direct measurement devices (using doppler effects) are still in their development stages. In fact, most streamflow series are estimated by the direct measurement of river stage (water level) and the use of a stage-streamflow relationship, known as a rating curve. As stated by Herschy (1995), "The operations necessary to develop the stage-discharge relation at a gauging station include making a sufficient number of discharge measurements and developing a rating curve by plotting the measured discharges against the corresponding stages and drawing a smooth curve of the relation between the two quantities." Finally, the continuous estimation of streamflow, for real-time purposes or to estimate a historical streamflow time series, is based on continuous river stage measurements and rating curve estimations.

However, streamflow estimations using stage measurements and rating curves are not exact. Errors in the stage-discharge relationship arise from three categories of uncertainties defined by Schmidt (2002, chap. 3):

natural uncertainties: inherent randomness of natural processes (turbulent fluctuation, wind, geometry of the section, and so on)
knowledge uncertainty: inadequate understanding of the physical processes

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data uncertainties: e.g., inaccurate measurement of water level or flow, inadequate spatial or temporal sampling.

The estimation of streamflow uncertainty becomes fundamental, as it could cause potentially great damage and large errors. For example, it could influence flood forecasting and security of infrastructures (dams, dikes), flood streamflow estimations (Clarke, 1999; Parodi and Ferraris, 2004; Petersen-Øverleir and Reitan, 2005; Moyeed and Clarke, 2005; Petersen-Øverleir and Reitan, 2009), flood and inundation stage estimations (Parodi and Ferraris, 2004; Shrestha and Simonovic, 2009; Pappenberger et al., 2006) and rainfall-runoff model calibration (Blazkova and Beven, 2009). Moreover, for a hydropower company, streamflow uncertainty estimation is also necessary to ensure the appropriate management of hydrometric stations, including gaugings planning, rating curve quality verification and streamflow quality assessment. Moreover, uncertainty estimation is also necessary to meet the requirements of rigorous environmental standards.

The initial uncertainty of rating curves, i.e., at time of establishment, has been well documented in the literature (Schmidt, 2002, chap. 3) for a relationship between stage and discharge in the form

$$Q = a(h - h_0)^b, \tag{1}$$

where Q is the discharge, h is the stage, h_0 is the stage of zero flow and a and b are form parameters. Generally, the parameters of Eq. (1) are estimated by regression analysis. Many methods have been developed to estimate the rating curve's uncertainty. Herschy (1995) used the standard error of the estimate and the standard error of the mean to assess the accuracy of the stage-discharge relationship in the form of Eq. (1). Clarke (1999) use the residuals variance in regression, including the uncertainty of the parameter h_0. In addition, Clarke et al. (2000) suggested using the residuals variance in regression to construct a confidence interval for discharge estimations. Moyeed and Clarke (2005) and Reitan and Petersen-Øverleir (2008) developed a method based on Bayesians statistics (Markov Chain Monte Carlo or MCMC) to adjust a rating curve on several gaugings and to construct a credibility interval for discharge estimations. Moreover, maximum likelihood methods have been proposed to estimate the parameters of Eq. (1) and the uncertainty of the rating curve (Venetis, 1970; Petersen-Øverleir, 2004; Petersen-Øverleir and Reitan, 2009) and multi-segment rating curve (Reitan and Petersen-Øverleir, 2008; Petersen-Øverleir and Reitan, 2005).

These approaches only estimate the uncertainty at the time of establishment of the rating curve, known here as *initial uncertainty*. However, depending on the geological nature of the watersheds and on hydro-meteorological events, the river bed can change over time, due to successive erosion and sediment deposit events. Typically, these processes can lead to non-stationary stage-discharge relationships, i.e., non-stationary rating curves. In the literature, the uncertainty caused by changes in river cross sections has received little attention (only a few words in Shrestha and Simonovic (2009), Petersen-Øverleir and Reitan (2009). In the case of gauging stations in mountainous areas, which are exposed to changes in river cross sections, the relationship between stage and discharge may change continuously over time and must be readjusted because the rating curve parameters are suitable only for a particular river cross section. The possible lack of representativeness between the rating curve and the actual stage-discharge relationship due to a river bed modification is considered as *temporal uncertainty*. Therefore, in these hydrometric situations, it is of primary importance to account for the probable changes in the cross section to estimate the streamflow uncertainty.

The aim of this article is to develop a method to estimate the temporal uncertainty of the assessed discharges from a rating curve. The method is based on a variographic analysis of a discharges time series. Usually, variographic analysis is used in geostatistics, but in this case it was adapted to estimate the temporal variance of a discharge prediction. The method was developed in an industrial context, where the objective is to apply this approach to a large number of various hydrometric stations. In order to ensure its application and its robustness, the method was applied to several hydrologic stations in the French Alps.

We present, at first, the description of the method (Section 2): the initial and temporal uncertainties of discharge that are estimated by the rating curve. Then, we apply the method to 19 hydrometrics stations (Section 3): description of the site, application on these stations and results. In the discussion (Section 4), the advantages and disadvantages of the method are discussed.

2. Methodology

2.1. Estimated discharges distribution

Intuitively, the uncertainty of estimated discharges should increase over time since the establishment of rating curve. Indeed, the risk of a change in the river bed increases with time. Mathematically, this situation is modeled by adding a component depending on time in the variance of the estimated discharge $\widehat{Q}(h,t)$ distribution:

$$\widehat{Q}(h,t) \sim \mathcal{N}\left(Q_{RT}(h), S_0^2(h) + \delta^2(h,t)\right), \tag{2}$$

where $Q_{RC}(h)$ is the estimated discharge from the rating curve at a stage h, $S_0^2(h)$ is the initial variance of the true value around the curve, t denotes the amount of elapsed time since the beginning of rating curve validity period and $\delta^2(h,t)$ represents what we call *aging* of the rating curve: the temporal uncertainty. We assumed independence when adding these variance components. This supposition is supported by the different underlying processes behind these components. In fact, initial variance $S_0^2(h)$ comes from natural, data and knowledge uncertainties, while the temporal uncertainty arises from modification of the river bed at the gauging site. In practice, independence of initial and temporal uncertainty is only an approximation. Indeed, since the rating curve estimation needs many gaugings, temporal processes occur during its calibration. A level of temporal uncertainty is thus included in the initial uncertainty. Also, when the river bed has changed, the subsequent errors are correlated which invalidate the assumption of independence. However, we suppose that both effects can be neglected without loosing much accuracy.

The normal assumption of estimated discharges is supported by the widespread use of regression analysis that was potentially used to estimate the parameters of the rating curve. In addition, a normal distribution of estimations has been used previously in the literature, for instance by Herschy (1995, chap. 14), where the variance was estimated by the square error of estimate (*see*) or by the square error of mean (*sem*), and by Moyeed and Clarke (2005) where the posterior distribution of the estimations was normal. However, as pointed out by Reitan and Petersen-Øverleir (2008), the normal distribution assigns a non zero probability for negative discharges, which can be problematic for drought discharges.

2.2. Initial uncertainty

When $t \to 0$, the initial uncertainty overcomes the temporal uncertainty. The initial uncertainty $S_0^2(h)$ can depend on the stage value because high discharges have larger variances. This variance