Journal of Hydrology 395 (2010) 91–102

Contents lists available at ScienceDirect

Journal of Hydrology

journal homepage: www.elsevier.com/locate/jhydrol

Reflection and refraction of obliquely incident elastic waves upon the interface between two porous elastic half-spaces saturated by different fluid mixtures

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ARTICLE INFO

Article history:
Received 22 May 2010
Received in revised form 30 August 2010
Accepted 6 October 2010

This manuscript was handled by L. Charlet, Editor-in-Chief, with the assistance of Chong-yu Xu, Associate Editor

Keywords:
Reflection
Refraction
Motional mode
Unsaturated porous medium

SUMMARY

A theoretical model for the analysis of the reflection and refraction of obliquely incident elastic waves upon the interface between two semi-infinite porous elastic half-spaces saturated by different fluid mixtures is developed in the present study based on the poroelasticity theory of Lo et al. (2005) and the normal coordinates derived by Lo et al. (2010) for describing the modes of dilatory motion. The amplitude and energy ratios of the reflected and refracted waves generated from either an incident P1 wave (the first dilatational wave) or an incident SV wave (the shear wave polarized in the vertical plane) are in turn theoretically determined for the first time with respect to the angle of incidence. As a representative example, a numerical simulation is conducted for Lincoln sand permeated by an air–water mixture in the lower half-space and Columbia fine sandy loam permeated by an air–water mixture in the upper half-space. Our numerical results indicate that regardless of the type of pore fluid mixtures and porous media, the sum of the energy ratio of the reflected and refracted waves is always equal to unity, a result that indeed can not be achieved if the normal coordinates for dilatory motional modes is not taken into account as to represent the Helmholtz potential of the reflected and refracted waves. In addition, their amplitude and energy ratios are shown to be significantly affected by the angle of incidence. It is also revealed that as a SV wave is incident upon the interface, a critical angle of 31° and 33° can be found for the reflected and refracted P1 waves respectively, while the occurrence of the critical angle is not observed for the case of an incident P1 wave.

1. Introduction

Over the past decades, the theoretical study on the reflection and refraction of obliquely incident elastic waves upon the interface between two fluid-containing poroelastic half-paces bas been of great interest to a variety of scientific fields, such as soil science, agriculture engineering, geomechanics, resource engineering, and hydrogeology (Deresiewicz, 1960; Deresiewicz and Rice, 1962, 1964; Sharma and Gogna, 1992; Roeloffs, 1996; Sharma, 2004; Arora and Tomar, 2007). A better understanding of quantitative information inferred from reflection and refraction wave characteristics is crucial for seismic methods in imaging the spatial distribution of hydrological and geological properties, such as permeability, porosity, and moisture variations in sedimentary materials, a sufficient level of which allows for the accurate description of water transport and contaminant movement in subsurface environments (Domenico, 1974; Geller and Myer, 1995; Bachrach and Nur, 1998; Adamo et al., 2004; Blum et al., 2004; Gorodetskaya, 2005).

An analytical model describing the reflection and refraction of elastic waves at the interface between two fluid-bearing porous media was first developed by Geertsma and Smith (1961), wherein wave incidence was presumed particularly to be in a normal direction. Deresiewicz and Levy (1967) studied the behavior of the reflected and refracted seismic waves in multiple layers of fluid-saturated porous media. An extension to a more general situation of an oblique incidence upon a plane interface was conducted by Hajra and Mukhopadhyay (1982). Wu et al. (1990) calculated the energy coefficient for the reflection and refraction of an obliquely incident wave upon the interface between a fluid and a fluid-permeated porous medium. The reflection and refraction of plane elastic waves at the loosely-bonded interface between an elastic solid and a fluid-containing porous medium were discussed by Vashisth et al. (1991), and then the energy ratio was depicted in terms of interface bonding constants. Sharma and Saini (1992) investigated the influence of pore alignment on the amplitude and energy ratios of the reflected and refracted waves at the interface between two different fluid-saturated poroelastic half-spaces. Lin et al. (2005) determined the surface displacement, surface strain, and energy partitioning for a plane wave reflecting in an inviscid fluid-saturated poroelastic half-space. Based on the work of Tuncay and Corapcioglu (1997) who showed three dilatational

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doi:10.1016/j.jhydrol.2010.10.018

waves (termed P1–P3) and one shear wave exist in an unsaturated porous medium, Tomar and Arora (2006) pioneered the study of the reflection and refraction of incident elastic waves for a two-fluid system in a porous half-space underlain by an elastic nonporous half-space. A numerical study later conducted by Arora and Tomar (2007) demonstrated that there is the existence of four reflected waves (P1–P3, and SV) and four refracted waves (P1–P3, and SV) when a P1 wave is incident upon the interface between two elastic porous half-spaces saturated by different fluid mixtures, but the effect of inertial coupling between solid and fluids was not taken into account in their study. In recent times, Arora and Tomar (2008) have employed the poroelasticity theory of Lo et al. (2005) where the effect of inertial coupling was systematically represented to examine its impact on the amplitude ratio of the reflected and refracted waves by comparing the results with those previously obtained by Arora and Tomar (2007).

Despite these advances, none of them, until now, have considered the existence of the normal coordinates for the solid and fluid dilatations, from which six connecting coefficients have been derived to specify their motional modes (Lo et al., 2010). In addition, it has been long demonstrated that three dilatational waves can be observed in an unsaturated porous medium (Santos et al., 1990; Tuncay and Corapcioglu, 1997; Lo et al., 2005), but the potential of these three waves was not completely represented in each potential of the reflected and refracted dilatational waves. In the present study, based on the general poroelasticity model of Lo et al. (2005) and the normal coordinates derived by Lo et al. (2010), the dynamics of the reflection and refraction of incident elastic waves upon the interface between two semi-infinite porous half-spaces containing distinct fluid mixtures was analytically examined in a systematic manner. To determine the amplitude and energy ratios of the reflected and refracted waves quantitatively, a numerical simulation is taken on for Lincoln sand saturated by an air–water mixture in the lower half-space and Columbia fine sandy loam saturated by an air–water mixture in the upper half-space as a function of the angle of incidence. Two different types of incident wave are investigated: the first dilatational wave (P1) and the shear wave polarized in the vertical plane (SV).

2. Model equations

A set of coupled partial differential equations of momentum balance for two-phase fluid flows in a deformable porous medium in the absence of body force was derived in an Eulerian framework, which takes the form (Lo et al., 2005):

$$\begin{aligned}\rho_s\theta_s\frac{\partial^2\vec{u}_s}{\partial t^2}&+A_{11}\left(\frac{\partial^2\vec{u}_1}{\partial t^2}-\frac{\partial^2\vec{u}_s}{\partial t^2}\right)+A_{12}\left(\frac{\partial^2\vec{u}_2}{\partial t^2}-\frac{\partial^2\vec{u}_s}{\partial t^2}\right)\\&+A_{21}\left(\frac{\partial^2\vec{u}_1}{\partial t^2}-\frac{\partial^2\vec{u}_s}{\partial t^2}\right)+A_{22}\left(\frac{\partial^2\vec{u}_2}{\partial t^2}-\frac{\partial^2\vec{u}_s}{\partial t^2}\right)+R_{11}\left(\frac{\partial\vec{u}_1}{\partial t}-\frac{\partial\vec{u}_s}{\partial t}\right)\\&+R_{22}\left(\frac{\partial\vec{u}_2}{\partial t}-\frac{\partial\vec{u}_s}{\partial t}\right)=\vec{\nabla}\left[\left(a_{11}+\frac{1}{3}G\right)\vec{\nabla}\cdot\vec{u}_s+a_{12}\vec{\nabla}\cdot\vec{u}_1\right.\\&\left.+a_{13}\vec{\nabla}\cdot\vec{u}_2\right]+\vec{\nabla}(G\vec{\nabla}\cdot\vec{u}_s),\end{aligned}\tag{1.1}$$

$$\begin{aligned}\rho_1\theta_1\frac{\partial^2\vec{u}_1}{\partial t^2}&-A_{11}\left(\frac{\partial^2\vec{u}_1}{\partial t^2}-\frac{\partial^2\vec{u}_s}{\partial t^2}\right)-A_{12}\left(\frac{\partial^2\vec{u}_2}{\partial t^2}-\frac{\partial^2\vec{u}_s}{\partial t^2}\right)-R_{11}\left(\frac{\partial\vec{u}_1}{\partial t}-\frac{\partial\vec{u}_s}{\partial t}\right)\\&=\vec{\nabla}(a_{12}\vec{\nabla}\cdot\vec{u}_s+a_{22}\vec{\nabla}\cdot\vec{u}_1+a_{23}\vec{\nabla}\cdot\vec{u}_2),\end{aligned}\tag{1.2}$$

$$\begin{aligned}\rho_2\theta_2\frac{\partial^2\vec{u}_2}{\partial t^2}&-A_{21}\left(\frac{\partial^2\vec{u}_1}{\partial t^2}-\frac{\partial^2\vec{u}_s}{\partial t^2}\right)-A_{22}\left(\frac{\partial^2\vec{u}_2}{\partial t^2}-\frac{\partial^2\vec{u}_s}{\partial t^2}\right)-R_{22}\left(\frac{\partial\vec{u}_2}{\partial t}-\frac{\partial\vec{u}_s}{\partial t}\right)\\&=\vec{\nabla}(a_{13}\vec{\nabla}\cdot\vec{u}_s+a_{23}\vec{\nabla}\cdot\vec{u}_1+a_{33}\vec{\nabla}\cdot\vec{u}_2),\end{aligned}\tag{1.3}$$

where ρ_α denotes the material density of phase α, the subscript α designating three immiscible phases: the solid ($\alpha = s$), the non-wetting fluid ($\alpha = 1$; fluid 1), and the wetting fluid ($\alpha = 2$; fluid 2); θ_α signifies the volumetric fraction of phase α and $\vec{u}_\alpha$ represents its displacement vector; R_{11} and R_{22} are the constitutive coefficients associated with viscous coupling between the solid and fluid phases; A_{12} and A_{21} are the constitutive coefficients pertinent to inertial coupling between the fluid phase and the adjacent fluid phase; A_{11} and A_{22} are those between the solid and fluid phases, the assumption of $A_{12} = A_{21}$ being typically made; G expresses the shear modulus of the porous framework; and a_{ij} $(i,j = 1,2,3)$ are the elasticity coefficients, and their cross terms are symmetric, i.e. $a_{ij} = a_{ji}$. The viscous coupling, inertial coupling, and elasticity coefficients can be determined in terms of directly-measurable parameters, a detailed discussion of which was given in Lo et al. (2005).

Let us consider a two-dimensional problem of elastic wave propagation and attenuation through two different porous elastic half-spaces saturated by various fluid mixtures in the plane x–z. The direction z points into the half-space and is positive downward. Thus, the Helmholtz potential of the displacement vector of solid and fluids along the directions x and z can be expressed as

$$u_\alpha=\frac{\partial\phi_\alpha}{\partial x}+\frac{\partial\psi_\alpha}{\partial z},\tag{2.1}$$

$$w_\alpha=\frac{\partial\phi_\alpha}{\partial z}-\frac{\partial\psi_\alpha}{\partial x},\qquad(\alpha=s,1,2)\tag{2.2}$$

where u_α and w_α represent the components of the displacement vector $\vec{u}_\alpha$ in the directions x and z, respectively, and $\vec{u}_\alpha$ is presumed to be independent of y; ϕ_α and ψ_α are two potential functions. It follows from Eqs. (2) that the dilatation and rotation of the α phase can be written as

$$\vec{\nabla}\cdot\vec{u}_\alpha=\frac{\partial^2\phi_\alpha}{\partial x^2}+\frac{\partial^2\phi_\alpha}{\partial z^2}=\nabla^2\phi_\alpha,\tag{3.1}$$

$$\vec{\nabla}\times\vec{u}_\alpha=\left(\frac{\partial^2\psi_\alpha}{\partial x^2}+\frac{\partial^2\psi_\alpha}{\partial z^2}\right)\hat{j}=\nabla^2\psi_\alpha\hat{j},\tag{3.2}$$

where $\hat{j}$ is the unit vector in the y direction. Next, taking the divergence of both sides of Eqs. (1) and then substituting Eqs. (2) and (3.1) into the result, we obtain

$$\begin{aligned}\rho_s\theta_s\frac{\partial^2\phi_s}{\partial t^2}&+A_{11}\left(\frac{\partial^2\phi_1}{\partial t^2}-\frac{\partial^2\phi_s}{\partial t^2}\right)+A_{12}\left(\frac{\partial^2\phi_2}{\partial t^2}-\frac{\partial^2\phi_s}{\partial t^2}\right)\\&+A_{12}\left(\frac{\partial^2\phi_1}{\partial t^2}-\frac{\partial^2\phi_s}{\partial t^2}\right)+A_{22}\left(\frac{\partial^2\phi_2}{\partial t^2}-\frac{\partial^2\phi_s}{\partial t^2}\right)\\&+R_{11}\left(\frac{\partial\phi_1}{\partial t}-\frac{\partial\phi_s}{\partial t}\right)+R_{22}\left(\frac{\partial\phi_2}{\partial t}-\frac{\partial\phi_s}{\partial t}\right)\\&=\left(a_{11}+\frac{4}{3}G\right)\nabla^2\phi_s+a_{12}\nabla^2\phi_1+a_{13}\nabla^2\phi_2,\end{aligned}\tag{4.1}$$

$$\begin{aligned}\rho_1\theta_1\frac{\partial^2\phi_1}{\partial t^2}&-A_{11}\left(\frac{\partial^2\phi_1}{\partial t^2}-\frac{\partial^2\phi_s}{\partial t^2}\right)-A_{12}\left(\frac{\partial^2\phi_2}{\partial t^2}-\frac{\partial^2\phi_s}{\partial t^2}\right)\\&-R_{11}\left(\frac{\partial\phi_1}{\partial t}-\frac{\partial\phi_s}{\partial t}\right)\\&=a_{12}\nabla^2\phi_s+a_{22}\nabla^2\phi_1+a_{23}\nabla^2\phi_2,\end{aligned}\tag{4.2}$$

$$\begin{aligned}\rho_2\theta_2\frac{\partial^2\phi_2}{\partial t^2}&-A_{12}\left(\frac{\partial^2\phi_1}{\partial t^2}-\frac{\partial^2\phi_s}{\partial t^2}\right)-A_{22}\left(\frac{\partial^2\phi_2}{\partial t^2}-\frac{\partial^2\phi_s}{\partial t^2}\right)\\&-R_{22}\left(\frac{\partial\phi_2}{\partial t}-\frac{\partial\phi_s}{\partial t}\right)\\&=a_{13}\nabla^2\phi_s+a_{23}\nabla^2\phi_1+a_{33}\nabla^2\phi_2.\end{aligned}\tag{4.3}$$