Journal of Hydrology 393 (2010) 370–380

Contents lists available at ScienceDirect

Journal of Hydrology

journal homepage: www.elsevier.com/locate/jhydrol

A sensitivity analysis of tide-induced head fluctuations in coastal aquifers

Luit J. Slooten [a,*], Jesus Carrera [b], Eduardo Castro [a], Daniel Fernandez-Garcia [a]

[a] Hydrogeology Group, Polytechnical University of Catalonia, Jordi Girona 1-3, Edificio D2, 08034 Barcelona, Spain
[b] Jaume Almera Institute, Lluís Solé Sabarí s/n, 08028 Barcelona, Spain

ARTICLE INFO

Article history:
Received 8 September 2009
Received in revised form 8 June 2010
Accepted 30 August 2010

This manuscript was handled by Philippe Baveye, Editor-in-Chief

Keywords:
Saltwater intrusion
Seawater intrusion
Groundwater
Sensitivity analysis
Calibration
Inverse problem

SUMMARY

The response of coastal aquifers to sea-level fluctuations, notably tides, is known to contain much information about hydraulic parameters. We performed sensitivity analyses to assess how much, about what and where this information can be best obtained. It is well known that the response to harmonic fluctuations (and many harmonics can be superimposed to describe sea-level fluctuations) decreases exponentially with distance inland. The characteristic length of this decay is $L_c = \sqrt{DP/\pi}$, where D is hydraulic diffusivity and P is period. Maximum sensitivity is obtained for distances equal to L_c, which is where maximum information would be obtained if the aquifer is treated as homogeneous. However, sensitivity depends not only on the problem dynamics, but also on parameterization. In fact, if heterogeneity is acknowledged by finely discretizing hydraulic conductivity, we find that connection to the sea (i.e. K near the coast) is what can be characterized best, while the most informative measurements are located at around 0.5 L_c. Thin low conductivity zones near the coast lead to a stepwise decrease in the amplitude of groundwater head fluctuations. We find that the fluctuations are independent of buoyancy effects, so that they can be simulated by constant density codes. High information content and ease of use suggest that they should be helpful in characterizing the aquifer–sea connection, which is important for coastal aquifer protection against seawater intrusion.

1. Introduction

Coastal aquifers can suffer seawater intrusion (SWI), which constrains the development of available groundwater resources. In particular, salinization of inland wells must be prevented. A critical issue controlling salinization is the degree of connectivity between the aquifer and the sea. Connectivity refers to the existence of spatially connected features that reduce travel time and increase flux. Unfortunately, connectivity is not easy to characterize with the data usually available in the field. Worse, flow connectivity indicators, which can be derived from hydraulic test data do not correlate well with transport connectivity indicators, which would control SWI (Knudby and Carrera, 2005). However, one of the most successful transport connectivity indicators is hydraulic diffusivity (Knudby and Carrera, 2006), which can be derived from fitting aquifer response to ocean tides.

The fitting can be performed in an inverse problem framework; e.g. (Carrera et al., 2005; McLaughlin and Townley, 1996; Cooley, 2004). In that case, the aquifer's flow and transport properties are expressed as a function of model parameters, which are estimated. The number of parameters that can be estimated and their uncertainty depends on how sensitive the tidal response is to these parameters. This can be quantified by means of sensitivity analysis (SA). The formal study of model sensitivity is increasingly advocated as a necessary step for understanding and using models and their predictions (Pappenberger and Beven, 2006). In its broadest sense, SA yields insights on which parameters control the system, what measurements are most informative, or what system's characteristics can be deduced from the observations of the system (Hill, 1998; Mehl and Hill, 2001; Chen and Chen, 2003). Unfortunately, SA carries a computational and interpretation cost several times greater than a simple simulation run, which may have discouraged its use for the study of models with many input parameters and long simulation running times. As discussed below, the use of SA can help in formulating a solvable inverse problem by identifying insensitive parameters. A parameter is insensitive when its value has little or no impact on model predictions; inversely, its value cannot be deduced accurately by fitting predictions to observations. Therefore, attempting to estimate it will lead to poorly posed inverse problems, whose overall convergence can be improved if the parameter is fixed to a reasonable value (Carrera and Medina, 1996).

The problem of parameter insensitivity has been studied both in general and specifically for SWI. In general, identifiability can be enhanced using different measurement types (Shoemaker, 2004); by modifying the formulation of the optimization problem to

* Corresponding author. Tel.: +34 934017247.
E-mail address: luitjan.slooten@upc.edu (L.J. Slooten).

0022-1694/$ - see front matter © 2010 Elsevier B.V. All rights reserved.
doi:10.1016/j.jhydrol.2010.08.032

include prior information (Carrera and Neuman, 1986) and a meaningful parameter a priori covariance model (Alcolea et al., 2006), or by gathering transient data sets that show how the system responds to external forcing (Sun and Yeh, 1992). Shoemaker (2004) used SA for evaluating the worth of several data types for aquifer characterization in a SWI setting, and concluded that the use of data of different types considerably reduces parameter estimation variance. Sanz and Voss (2006) studied the model sensitivity of the Henry problem (Henry, 1964), an abstraction of a coastal aquifer with a saltwater intrusion wedge in steady state.

When calibrating SWI models, the above mentioned measures to reduce insensitivity can certainly be applied. Interaction with the sea facilitates complementing hydraulic head data with salinity measurements. The intrusion shape, penetration depth and the width of the mixing zone are sensitive both to flow and to transport parameters and provide information at a geographical scale far larger than that of ordinary tracer tests. Moreover, ocean tides create a periodic external forcing that may increase further the identifiability of the flow parameters.

However, the processes involved (tidal wave propagation, seawater intrusion and the effects of heterogeneity) have been studied largely separately, or in groups of two. Overviews of SWI dynamics and modeling are given in Bear et al. (1999), Diersch and Kolditz (2002). Analytical and numerical modeling of the propagation of tidal waves in aquifers can be found in e.g. Carr and Vanderkamp (1969), Townley (1995), Trefry (1999) and Li and Jiao (2001) (focusing on flow), and e.g. Yim and Mohsen (1992), Bolster et al. (2007) and Elfeki et al. (2007) (focusing on transport). Tidal dynamics in unconfined aquifers have been studied in e.g. Jeng et al. (2005) and Teo et al. (2003).

Heterogeneity is probably the most ubiquitous property of aquifers. Spatial variability of K controls water flow and solute transport. SWI should be no exception, but has received scant attention (with the exception of e.g. Abarca (2006) and Held et al. (2005), who analyzed the effect of heterogeneity on the shape of the SWI wedge). (Alcolea et al., 2007) solved the inverse problem in a heterogeneous aquifer using tidal wave driven head fluctuations, but assuming constant water density. Brovelli et al. (2007) studied contaminant transport in a homogeneous aquifer with SWI and tides. The interaction between flow fluctuations and spatial variability of K has been studied by Dentz and Carrera (2003, 2005) and Cirpka and Attinger (2003), who evaluated the resulting increase in dispersivity.

While these works have provided useful insights on the dynamics of coastal aquifers, there has been relatively little focus on sensitivity analysis, especially regarding tides and heterogeneous media. Still, the issue is important both for the design of observation networks and for finding a suitable model parameterization. Therefore, in this paper the effect of the presence of tides on parameter sensitivity is studied with the ultimate aim of identifying optimal monitoring locations and the parameters that can be obtained from measurements of tidal response. The key of our approach is the separation of effects that different simplifications and assumptions have. To illustrate this, consider a heterogeneous flow domain. In such a domain, the observed sensitivity will be different from that in a homogeneous domain. However, in order to represent a heterogeneous domain, a fitting parameterization must be chosen. So when sensitivity is different: what part of the difference can be attributed to heterogeneity and what part to a different parameterization? To better separate the impact of different simplifications and assumptions in the mathematical model, we start with a very simplified model (one-dimensional constant density flow) and parameterize it in different ways. Next, two-dimensional constant density flow problems are studied, with an without tides and with different parameterizations. Each time, SA is performed and the results are compared. Finally, two-dimensional variable density flow and transport is studied in the same way. For this last case, a variation on the classical Henry problem was used: the anisotropic and dispersive Henry problem proposed by Abarca et al. (2007), which more closely resembles a real SWI (Henry exaggerated molecular diffusion many orders of magnitude).

In all these cases (1d flow, 2d flow and 2d with variable density flow and transport), SA was carried out for three parameterizations of the hydraulic conductivity. In the first parameterization, a single parameter represents hydraulic conductivity in all the domain. In the second parameterization, hydraulic conductivity is computed by kriging over a set of point values of K. These point values are the parameters, but they all have the same value, so that the aquifer is in fact homogeneous. The third parameterization differs from the second in that the parameters now have different values, making hydraulic conductivity effectively heterogeneous. Flow and transport were solved with the finite element method; sensitivity to parameters was based on analytical derivatives rather than on the finite difference approach commonly applied. This method makes SA computationally feasible for SWI problems even when using hundreds of parameters.

2. Background

2.1. Astronomical tides

Variations in surface water height in seas and oceans are mainly caused by wind, variations in atmospheric pressure, and ocean tides (Pugh, 1987). While the first two are irregular, the tide component is periodic and predictable. Tides are caused by variations in the gravitational pull from the celestial bodies, the most important of these being the moon (for its proximity) and the sun (for its large mass). Mathematically, tides can be accurately described by a number of harmonic terms of the form $A \cdot \cos(\sigma t - g)$ where A is an amplitude, σ an angular velocity and g a time lag. These constants are calibrated locally around the world to account for factors such as the connectivity with the open ocean and basin morphology. This leads to a number of "harmonic constituents" (Doodson, 1921) which can be obtained for particular locations from oceanographic institutes worldwide (see Table 1). Most components are either semidiurnal (around 12 h period) or diurnal (around 24 h period), although there are some small monthly and annual components. In most places the most important ones are the lunar M2 and the solar S2 components, both semidiurnal. The small difference in period between these two is the main cause of the monthly cycle of tides.

In the Mediterranean, tidal amplitudes vary from a few centimeters to decimeters; in the North Sea, values between 1 m and 3 m are typical. The largest tides of the world are in the bays of Ungava and Fundy, both in Canada (around 13 m of mean spring tide range).

2.2. Sensitivity

Model sensitivity refers generically to the relationship between model input and output. There are different approaches to sensitivity computation. The most used measure is the partial derivative of an output value u_i to an input parameter p_j (Eq. (1)), the calculation of which is implemented by popular parameter estimation packages such as UCODE (Poeter et al., 2005) and PEST (Doherty, 2001).

$$X_{ij} = \frac{\partial u_i}{\partial p_j} \tag{1}$$

where X_{ij} denotes the sensitivity of u_i to p_j. It is local, in the sense that it is evaluated for a certain parameter vector. As such, the matrix X is a complete description of model sensitivity only if model output is a linear function of input parameters. Otherwise, the local