Journal of Hydrology 389 (2010) 111–120

Contents lists available at ScienceDirect

Journal of Hydrology

journal homepage: www.elsevier.com/locate/jhydrol

Multifractal analysis of African monsoon rain fields, taking into account the zero rain-rate problem

S. Verrier *, L. de Montera, L. Barthès, C. Mallet

Université de Versailles Saint-Quentin-en-Yvelines, CNRS/INSU, LATMOS – Laboratoire Atmosphères, Milieux, Observations Spatiales, 11 boulevard d'Alembert, 78280 Guyancourt, France

ARTICLE INFO

Article history:
Received 23 July 2009
Received in revised form 26 March 2010
Accepted 19 May 2010

This manuscript was handled by A. Bardossy, Editor-in-Chief, with the assistance of P. Chandra Nayak, Associate Editor

Keywords:
Precipitation
Multifractals
Radar data
Zero rain rates
AMMA campaign

SUMMARY

Nonlinear rain dynamics, due to strong coupling with turbulence, can be described by stochastic scale invariant (such as multifractal) models. In this study, attention is focused on the three-parameter fractionally integrated flux (FIF), based on the universal multifractal (UM) model developed by Schertzer and Lovejoy (1987). Multifractal analysis techniques were applied to experimental radar data measured during the African monsoon multidisciplinary analysis (AMMA) campaign, during the summer of 2006. The non-conservation parameter *H*, which has often been estimated at 0, was found to be more likely close to 0.4, meaning that rain is not a conserved cascade. Moreover, it is shown that the presence of numerous zero values in the data has an influence, which has until now been underestimated, but should in fact be accounted for. UM parameters are therefore estimated from the full dataset, and then only from maps in which almost all pixels have a non-zero value. Significant differences were found, attributed to *on–off* intermittency, and their role was checked by means of simulations. Finally, these results are compared with those previously based on time series, and collected by a co-localized disdrometer. The sets of parameters obtained in the spatial and time domains are found to be quite close to each other, contrary to most results published in the literature. This generally reported incoherency is believed to result mainly from the influence of *on–off* intermittency, whose effects are stronger for time series than for selected radar maps.

1. Introduction

When modeling the atmosphere, one needs to take into account various significant fields, such as wind, temperature, humidity or rain rate, as well as their coupling interactions. Due to its highly nonlinear and intermittent behavior, the rain rate field remains one of the most difficult to model. Phenomenological models and statistical tools are generally required. Rain is strongly coupled with atmospheric turbulence, which in the inertial range can be statistically described by scale invariant processes, due to the fundamental symmetries of the Navier–Stokes equations. These scale invariant processes do not have any characteristic scale over the scaling range. Moreover, interactions occur preferentially between neighbouring scales (localness in Fourier space), and energy fluxes may be conserved from large to small-scales. The latter three properties lead to a cascade phenomenology. To illustrate this, we recall the classical Kolmogorov and Corrsin–Obukhov scaling laws (Kolmogorov, 1941; Obukhov, 1949; Corrsin, 1951), which respectively describe longitudinal velocity (v) and passive-scalar (ρ) field increments (denoted Δv and $\Delta\rho$). These statistical laws are based mainly on the assumption of energy flux density (ε) and scalar variance flux density (χ) conservation, as well as on dimensional considerations. Their expressions, in the case of homogeneity of the fluxes, are respectively:

$$\Delta v \approx \varepsilon^{1/3} l^{1/3}$$

$$\Delta\rho \approx \chi^{1/2} \varepsilon^{-1/6} l^{1/3}$$

where l is the scale at which increments are considered.

The case of inhomogeneous fluxes was investigated later (Kolmogorov, 1962; Obukhov, 1962): although they have a constant average, scale-by-scale transfers may be intermittent. Scaling cascade models were then proposed in order to represent such intermittency (Novikov and Stewart, 1964; Yaglom, 1966; Mandelbrot, 1974). Turbulence (and rain) models were therefore adapted to reproduce scale invariance properties through the use of fractal geometry.

However, it was found that a single fractal dimension was insufficient to describe the whole phenomenon, such that more sophisticated models were developed in the 1980s, based on multifractal formalism. In these models, rain is described by an infinite set of

* Corresponding author.
E-mail address: verrier@latmos.ipsl.fr (S. Verrier).

0022-1694/$ - see front matter © 2010 Elsevier B.V. All rights reserved.
doi:10.1016/j.jhydrol.2010.05.035

fractal dimensions, each of these being associated with a singularity level.

Then, it was shown by Schertzer and Lovejoy (1987) that multifractal rain properties could be reproduced by a multiplicative cascade process. In the same paper, the authors mention the stability properties of cascade generators, and argue the existence of a class of attractors (called universal multifractals (UM)). Moreover, since geophysical processes are generally non-stationary, an additional fractional integration is needed. Such integrated cascades are referred to as fractionally integrated flux (FIF). This model is described by only three fundamental parameters (Schertzer and Lovejoy, 1991): a multifractality exponent α, an average sparsity degree C_1, and a non-conservation parameter H.

Various studies, based on this or similar formalisms, have shown the pertinence of multifractal rain models: (Lovejoy et al., 1987; Lovejoy and Schertzer, 1990; Gupta and Waymire, 1991; Ladoy et al., 1991; Tessier et al., 1993; Hubert et al., 1993; Olsson, 1995; Olsson and Niemczynowicz, 1996; Tessier et al., 1996; Marsan et al., 1996; de Lima, 1998; Hogan and Kew, 2005; Lovejoy and Schertzer, 2006; Lovejoy et al., 2008). Typically, the parameters are estimated as ($\alpha = 0.6$, $C_1 = 0.5$, $H = 0$) in time, and ($\alpha = 1.4$, $C_1 = 0.15$, $H = 0$–0.3) in space. For a review of this topic, see Lovejoy and Schertzer (1995) and Lilley et al. (2006). However, a recent study (de Montera et al., 2009), which extends the findings of previous papers (Harris et al., 1996; Schmitt et al., 1998), has shown that the estimation method is biased due to the high number of zero values present in the data (in the following, the fluctuation of the cascade level is classically referred to as 'intermittency', whereas the alternation between rain and no-rain periods is called *on-off* intermittency). Indeed, the analysis of *uninterrupted* rain rate time series has led to significantly different parameter estimations ($\alpha = 1.7$, $C_1 = 0.13$, $H = 0.53$).

Following a similar approach, the purpose of the present study is to estimate biases provoked by *on–off* intermittency in the spatial domain, and to perform a new estimation of multifractal parameters based on radar rain maps. In particular, in order to estimate more reliable spatial parameters, we focus on rain map subsections in which almost all pixels have a non-zero value (called *full-rain* maps in the following). Firstly, the essential equations of the multifractal framework and the multifractal analysis are recalled (Sections 2 and 3). The experimental datasets are presented in Section 4. Then, Section 5 presents the results of the multifractal analysis performed both on classical rain maps, and on *full-rain* sub-sections. The influence of the presence of zero values is verified by numerical simulations, and the differences between the parameters are thus interpreted (Section 6). Finally, the new set of unbiased universal parameters is compared with those proposed by de Montera et al. (2009), and in other scientific publications (Section 7).

2. Properties of multifractals

In this section, the fundamental properties of fractals and multifractals are recalled. In particular, we introduce the FIF model used in the following sections.

2.1. Fractal sets and multifractal fields

The concept of fractal dimension is useful to characterize scale invariance in the context of geometric sets (for a detailed introduction, see Falconer (2003). We define a fractal set A embedded in a space of dimension D, and of characteristic size L. By considering A on a scale defined by l, equivalent to a resolution of $\lambda = L/l$, the embedding space is divided into λ^D boxes B_λ of size l^D. Each box may, or may not belong to the fractal set A, seen at a resolution λ (denoted A_λ). The associated probability is:

$$\Pr(B_\lambda \in A_\lambda) \approx \lambda^{-C_f} \tag{1}$$

where $C_f = D - D_f$ is the fractal co-dimension of A, and D_f is its (box-counting) fractal dimension (here and in the following, $\approx$ indicates an equality, within the limits of slowly varying functions).

In the case of rain, the situation is more complex than in a binary set because we are interested in modeling not only its presence or absence, but also its intensity. Therefore, multifractal models must be used instead of monofractal models with a single co-dimension. In order to define a threshold independently of the resolution, the scale invariant notion of singularity γ is used. The singularity is related to the threshold T by the relation:

$$T = \lambda^\gamma \tag{2}$$

For any singularity, a family of exceedance sets may be obtained, each set being associated with a specific resolution. The fundamental equation of the multifractal formalism is then:

$$\Pr(\phi_\lambda > \lambda^\gamma) \approx \lambda^{-c(\gamma)} \tag{3}$$

where ϕ_λ is the field seen at resolution λ, and $c(\gamma)$ is the specific co-dimension corresponding to the set defined by the singularity γ. Due to the equivalence relation between probability distributions and statistical moments, (3) may also be written (Schertzer and Lovejoy, 1987) as:

$$\langle \phi_\lambda^q \rangle \approx \lambda^{K(q)} \tag{4}$$

where $\langle \bullet \rangle$ is the averaging operator, q is the order of the moment, and $K(q)$ is the so-called moment scaling function which characterizes the multifractal field. In the general case, $K(q)$ is a convex function with the trivial special values $K(0) = 0$ and $K(1) = 0$. Eq. (4) means that for any given order q, the moment depends on the resolution through a simple power-law. It can be shown that there is a one-to-one correspondence between singularities and moment orders, since the moment scaling function is the Legendre transform of the co-dimension function (Parisi and Frisch, 1985). Two multifractal fields with the same moment function are equivalent, in the sense of equivalence classes, although they can be physically very different (for more details concerning the properties of the multifractal fields, see Schertzer et al. (2002)).

2.2. Multiplicative cascades

Multiplicative cascades, first developed in the framework of turbulence theory, may be used to build multifractal fields. The main idea is to start the cascade at larger scales and to derive the process at smaller scales, by successively dividing pixels into sub-pixels, and determining the value of each smaller pixel from that of the larger ones, through multiplication by i.i.d. random variables, independently of the scale. The simplest multifractal case is the α-model (Schertzer and Lovejoy, 1985) in which the random variable has two possible values, defining multiplicative weights that respectively correspond to an increasing or a decreasing pixel value. Usually, as for turbulence in which the energy flux is conserved by the nonlinear terms of the Navier–Stokes equations, the mean of the process is assumed to be conserved when the resolution changes:

$$\forall \lambda, \langle \phi_\lambda \rangle = M \tag{5}$$

In the following, we assume that ϕ_λ is a normalized conservative process, such that $M = 1$.