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Valuing hydrological forecasts for a pumped storage assisted hydro facility

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SUMMARY

This paper estimates the value of a perfectly accurate short-term hydrological forecast to the operator of a hydro electricity generating facility which can sell its power at time varying but predictable prices. The expected value of a less accurate forecast will be smaller.

We assume a simple random model for water inflows and that the costs of operating the facility, including water charges, will be the same whether or not its operator has inflow forecasts. Thus, the improvement in value from better hydrological prediction results from the increased ability of the forecast using facility to sell its power at high prices. The value of the forecast is therefore the difference between the sales of a facility operated over some time horizon with a perfect forecast, and the sales of a similar facility operated over the same time horizon with similar water inflows which, though governed by the same random model, cannot be forecast.

This paper shows that the value of the forecast is an increasing function of the inflow process variance and quantifies how much the value of this perfect forecast increases with the variance of the water inflow process. Because the lifetime of hydroelectric facilities is long, the small increase observed here can lead to an increase in the profitability of hydropower investments.

Introduction

What is the value of the forecast of an environmental variable, such as temperature, wind speed, or water inflow rate, for some point? This is a question which has intrigued people for many years. Such forecasts have value to companies active in many different industry sectors, including transportation, leisure, and energy. In the electrical power industry, temperature is a crucial variable, the value of which to providers of electrical "spinning reserve" has been quantified in Hobbs et al. (1999). For producers of hydroelectricity, water inflow and water level in various points is of great interest. The value of long range forecasts to operators of hydro dams on the large Columbia River watershed was estimated in Hamlet et al. (2002).

This paper provides a theoretical framework for investigating the value of much shorter lead time hydrological forecasts to hydro producers, operating both with and without pumped storage facilities, on a small watershed. The operator of such a facility faces hydrological inflows that are somewhat uncertain even on the 48 h time scale. Improved hydrological forecasts can add value to hydro power producers in two ways: by improving their ability to forecast prices and to improve their ability to manage variable water inflows. This paper focuses on the latter, volumetric uncertainty, aspect and assumes that the operator is certain of the price for electrical power over the same 48 h time scale, either because the plant operates in a regulated electricity market or because accurate short-term price forecasts are available.

To compute V_d, the sales of a facility operated over some time horizon with a perfect forecast, we use Monte Carlo techniques to find the average best practices sales with forecast information. To do this we first simulate a water inflow time series from our hydrological model. We then use dynamic programming to find the optimal way for a model hydro plant, operating with water level constraints, to use this water for power generation and the optimal sales level which results. The value of V_d is the average over many realizations of this process.

V_r, the sales of a similar facility operated over the same time horizon with similar water inflows which cannot be forecast, is computed using stochastic dynamic programming to find the optimal operation protocol for the same power plant with the same constraints exposed to the same random inflow process. V_r is the expected sales of the plant operated in this optimal way.

The structure of the paper is as follows: "Difficulties inherent in hydrological modelling" briefly summarizes the many difficulties inherent in hydrological modelling and describe our simple model for random water inflows. "Modelling of hydroelectric facility" introduces modelling of hydroelectric facility and also describe the pumped storage facility model. "Optimization algorithm" focuses on the programming algorithm that is used to determine

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the optimal operation. "Modelling of hydroelectric facility" and "Optimization algorithm" draw heavily on the earlier work of Thompson et al. (2004) and Zhao and Davison (2009). "Analysis of results" describes the numerical experiments conducted and the results of these calculations. "Value of water inflow forecast" analyzes the value of hydrological forecast for a pumped storage facility. The last section presents conclusions and suggestions for future work.

Difficulties inherent in hydrological modelling

A hydrological forecast for a given region can be undertaken with a model for forecasting precipitation over a given watershed. Precipitation is delivered to streams both as overland flow to tributary channels and by subsurface flow rates as groundwater (Freeze and Cherry, 1979). Determining where and how the water travels, as overland flow, on the terrain requires an understanding of hydraulics. A detailed model also requires accurate topographic information (which can be obtained from various data sources, such as digital elevation models, and then entered into Geographical Information Systems (GIS)).

In principle, once this hydrologic information is integrated, a computationally intensive process can be used to make predictions of water levels (referenced to some datum such as sea level) and flow rates. Water level information can then be used to calculate the volume stored in each surface mass of water within a GIS framework. Given the inherent difficulty of making accurate point forecasts of rainfall, this approach is very challenging to implement.

The other missing piece of the puzzle is measuring the amount of time it takes for a raindrop to travel from where it fell to another point on Earth's surface. This travel time is called the "characteristic time" and, with this measurement in hand, the flow and level at this point can be obtained by "looking backwards" and determining the rainfall at that time (either by forecasting or by looking at observed data of past rainfalls). It should be noted that even this "backcasting" step, if applicable, is not trivial as detailed fine grained rain measurements are rarely made throughout a watershed, let alone recorded.

The forecasting step is even more challenging, because the grid- or mesh-size on a General Circulation Model (GCM) is much larger than the area over which rainfall forecasts are needed. However, the prediction of rainfall is very difficult. Rainfall is extremely variable on spatial and temporal scales, with scaling reported to be fractal (Breslin and Belward, 1999; Rajagopalan and Tarboton, 1993). It is created by a variety of interacting physical processes which occur on scales varying from micrometers to hundreds of kilometers (Rajagopalan and Tarboton, 1993). As weather models have a grid size of just a few kilometers, physical processes, including the important ones relating to cloud formation occurring on smaller length scales must be approximated, for instance by the "dynamical downsizing" of these grids (Cluckie et al., 2006).

Even though there are many challenges in precipitation forecasting and the difficulty of modelling watersheds (Krzysztofowicz and Herr, 2001; Krzysztofowicz et al., 1993), there are some available streamflow-based models, which capture the random characteristics of the adjusted historical streamflows data, able to perform sufficiently accurate inflow predictions (Wurbs, 1993). For a case study of how hydrological inflow forecasts are used by hydroelectric operators, see Druce (1990) and Druce (1994).

In this paper, we create a simple random model for a hourly water flow into the upper reservoir of a pump-assisted storage facility. We optimize the value of the facility, run for the next 48 h, in the face of this random inflow. Then, for 100 different realizations of this random inflow process, we optimize the value of the same facility with the inflows known for the next 48 h. The average of these values, which is equal to the uncertainty optimized value, is less than the deterministic value—we quantify the exact difference in "Value of water inflow forecast".

Modelling of hydroelectric facility

There are two main categories of hydroelectric power generation: conventional methods (dams and run-of-the-river), which produce electricity via water flow in one direction; and pumped storage methods, which are both producers and consumers of electricity.

The optimal operation of hydroelectric generation facilities depends on the price of power, p, inflow rate, f, the total amount of water, w, and the power function, $\tilde{E}$, as discussed in Thompson et al. (2004) and Zhao and Davison (2009). We seek the controlling flow rate, c, to maximize the cash value, $\tilde{V}(t,T,p,w)$, which is defined as

$$\begin{cases} \tilde{V}(t,T,p,w) = \max_c \mathbb{E}\left[\int_t^T e^{-r(s-t)}p(s)\tilde{E}(c,w)ds + e^{-r(T-t)}\tilde{R}(w(T))\right], \\ dw = \mu(f,c)ds + \sigma(f)dB_s. \end{cases} \quad (1)$$

Here, $\mu(f,c)$ is the mean and $\sigma(f)$ is the variance, of water amount, t and T are the beginning and ending time, r is a discount factor for the time value of money, $\tilde{R}(w_T)$ is the residual value of the water remaining in the reservoir at the end of the time horizon, and B is Brownian motion.

Because the reservoir cannot be drained below some minimum water level nor filled above some maximum level, the amount of water stored is constrained to lie within $w \in [w_{min}, w_{max}]$, and the release flow c is also constrained (depending on w) to lie in the range $c \in [c_{min}(w), c_{max}(w)]$.

The theoretical power P available from a given head of water is in direct proportion to the head h and the release rate c (Hydropower basics, 2007). The output power of hydro–turbo generations is a function of both the net hydraulic head and the water discharge (El-Hawary and Christensen, 1979). If P is measured in Watts, c in m^3/s and h in meters, $\rho = 1000\ kg/m^3$, g is the acceleration due to gravity m/s^2, the gross power of the flow of water is: $P = \rho gch$. When we release or pump water at $c\ m^3/s$ (i.e. $3600c\ m^3/h$) at water head h, the water head changes at the rate dh, $dh = c/S$, where S is the bottom area of the reservoir. To simplify the numerical details of the computation we henceforth assume that the reservoir is cuboid and $S = 3600\pi\ m^2$. In this case if we release water at rate $c\ m^3/s$, $dh = c/\pi$ m/h, and our computations are simplified by using the water head h rather than water volume w. Suppose $dh = (f - c)/Sdt + \sigma f/S\ dB_t$ and rewrite the objective function (1), so,

$$\begin{cases} V(t,T,p,h) = \max_c \mathbb{E}\left[\int_t^T e^{-r(s-t)}p(s)E(c,h)ds + e^{-r(T-t)}R(h(T))\right], \\ dh = (f-c)/Sds + \sigma f/SdB_s. \end{cases} \quad (2)$$

Discretize objective (2) as

$$\begin{cases} V(t,T,p,h) = \max_c \mathbb{E}\left[\sum_{s=t}^{T} e^{-r(s-t)}pE\Delta s + e^{-r(T-t)}R(h(T))\right], \\ h_{s+\Delta s} = h_s + (f_s - c_s)/S\Delta s + \sigma f_s/S\Delta B_s. \end{cases} \quad (3)$$

In particular, we assume that the turbine efficiency η is given by function (4) as described in Thompson et al. (2004),

$$\eta(c,h) = -\eta_{max}\left(\frac{10^{-6}\rho gch}{\psi} - 1\right)^2 + \eta_{max}. \quad (4)$$