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Hydrograph separation for karst watersheds using a two-domain rainfall–discharge model

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SUMMARY

Highly parameterized, physically based models may be no more effective at simulating the relations between rainfall and outflow from karst watersheds than are simpler models. Here an antecedent rainfall and convolution model was used to separate a karst watershed hydrograph into two outflow components: one originating from focused recharge in conduits and one originating from slow flow in a porous annex system. In convolution, parameters of a complex system are lumped together in the impulse-response function (IRF), which describes the response of the system to an impulse of effective precipitation. Two parametric functions in superposition approximate the two-domain IRF. The outflow hydrograph can be separated into flow components by forward modeling with isolated IRF components, which provides an objective criterion for separation. As an example, the model was applied to a karst watershed in the Madison aquifer, South Dakota, USA. Simulation results indicate that this watershed is characterized by a flashy response to storms, with a peak response time of 1 day, but that 89% of the flow results from the slow-flow domain, with a peak response time of more than 1 year. This long response time may be the result of perched areas that store water above the main water table. Simulation results indicated that some aspects of the system are stationary but that nonlinearities also exist.

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Introduction

Rainfall–discharge (or rainfall–runoff) models relate precipitation on a watershed to the resulting streamflow or spring flow from the watershed (Jakeman and Hornberger, 1993; McDonnell et al., 2007). A physically based approach to simulating rainfall–discharge relations in karst watersheds can be difficult because detailed and quantifiable information about the physical parameters (soil, vegetation, fractures, conduits, swallow holes) often is not available. Highly parameterized watershed models sometimes contain more information than is necessary and result in little or no improvement over simpler, less parameterized models (Jakeman and Hornberger, 1993). In these cases, large uncertainties in parameter estimates may result (Doherty, 2007). McDonnell et al. (2007) assert that exploration of the organizing principles and processes that underlie watershed heterogeneity and complexity is preferable to explicit characterization of watershed heterogeneity in "highly calibrated" models. This paper describes a modeling approach that helps characterize these organizing principles and processes.

Convolution is the integration of the product of a forcing function or signal and a time lagged impulse-response function that results in a system response (Jenkins and Watts, 1968), and is effective for simulating rainfall–discharge relations (Singh, 1988; Olsthoorn, 2008). Dreiss (1989) applied this method to karst watersheds. Because this approach is not physically based, it can be applied with few parameters. In this method, effective precipitation is the signal that results in a response in watershed outflow, and an instantaneous unit hydrograph or impulse-response function (IRF) describes the system's response characteristics. The response times from a single impulse can be widely spread over time, and the IRF is an estimate of the full spectrum of these response times. The IRF operates on the signal by convolution to translate the signal into the response function; i.e., spring flow or streamflow. If the IRF is assumed to fit a parametric function, few parameters are needed, which may increase model confidence as measured by statistical metrics in inverse modeling. In highly porous karst watersheds where overland runoff is negligible, watershed outflow results primarily from spring flow or groundwater fed streamflow. The distinct domains of quick and slow flow in karst settings have been represented by an IRF that is a combination of two or more functions (Jakeman et al., 1990,1993; Post and Jakeman, 1996; Pinault et al., 2001; Denic-Jukic and Jukic, 2003; Long and Putnam, 2004, 2006), which allows the characteristics of each of these domains to be quantified separately.

Traditional methods for graphical hydrograph separation assume that watershed outflow is a binary system separated into

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base flow and stormflow components. These methods, which were summarized by Sloto and Crouse (1996), were based on the streamflow hydrograph itself with no other data requirements. Different hydrograph separation methods and different choices of parameter values result in different estimates of the division between base flow and stormflow, indicating that this estimate is, in most cases, subjective to some degree. Recent hydrograph separation methods that combine rainfall, runoff, and tracer data (e.g., Weiler et al., 2003) are an improvement over earlier methods. Although precipitation data are readily available in many cases, tracer data are less abundant.

In many karst watersheds, subsurface flow emerging as surface outflow is a mixture of fast pipe flow in large conduits and slower flow in an annex-system of smaller passages, such as fractures and vugs (Palmer, 1991; Mangin, 1994; Kaufmann and Braun, 2000). Mixing of these two domains (i.e., quick flow and slow flow) may occur to some degree before discharging to the stream. The distinction between these flow domains with respect to the streamflow hydrograph is presented in this paper as an objective criterion to separate the hydrograph when tracer data are not available.

This paper presents a method for determining the two-domain IRF for karst watersheds using an antecedent rainfall and convolution model based on precipitation and spring flow or streamflow data. Once parameters are estimated, hydrograph separation is performed by forward modeling with isolated components of the IRF. This method also provides an estimation of the distribution of response times to an impulse as well as information about antecedent rainfall effects, which is useful for characterization, quantification, and comparison of different watersheds. Because stationarity frequently is assumed for convolution, the effects of this assumption were assessed. A simple method to account for snow accumulation and melting is presented. As an example, the model was applied to a perennial karst watershed, where flow into the channel from the subsurface occurs throughout much the watershed as streamflow. Previous applications of two-domain IRFs for rainfall–discharge models have resulted in IRF components that mostly overlap. Others use separate functions for the rising and declining limbs or for the peak and tail in a composite function (e.g., Denic-Jukic and Jukic, 2003). However, some karst catchments may require a slow-flow IRF component that peaks much later than that of quick flow. An application is presented where this might be the case, and an exponential function for slow flow is tested against a lognormal function. Inverse modeling provided an efficient means of estimating parameters and assessing model uncertainty.

The model

Effective precipitation

Moisture content of the soil in a watershed is an important factor for determining the response of spring flow or streamflow to precipitation. The amount of precipitation falling on a watershed that ultimately becomes outflow is referred to as effective precipitation. This includes infiltration of precipitation that emerges as outflow and direct runoff. The amount of rain that has fallen prior to a particular rainfall event—the antecedent rainfall condition—has a large influence on soil moisture content. Heavy rains prior to an event will saturate the root zone and cause a decrease in evapotranspiration. Higher humidity and cooler temperatures during wetter periods also may decrease evapotranspiration rates. The method of Jakeman and Hornberger (1993) calculates an antecedent rainfall index s_i, which weights the daily rainfall by previous rainfall. The weighting s_i is distributed exponentially backward in time by

$$\begin{aligned} s_i &= cr_i + (1-\kappa^{-1})s_{i-1} \\ &= c\lfloor r_i + (1-\kappa^{-1})s_{i-1} + (1-\kappa^{-1})^2 s_{i-2} + \cdots \rfloor \quad i \\ &= 0,1,\ldots,N \; 0 > s_i > 1, \end{aligned} \tag{1}$$

where c is a normalizing parameter to limit s_i to values between 0 and 1 (dimensionless), κ adjusts the influence of antecedent conditions (dimensionless), r_i is total daily rainfall (cm), and i is the time step in days. Effective daily precipitation u_i (cm) is then calculated by

$$u_i = r_i s_i. \tag{2}$$

In cold climates, snowfall accumulates during winter and early spring and results in little infiltration until melting occurs. Melting snow can result in considerable infiltration and a high antecedent rainfall index for the spring rains that follow. Sublimation during winter reduces the amount of snow available for spring infiltration. In these cases, the measured winter precipitation (in centimeters of water) can be redistributed to the spring melting period in the model and then added to the rainfall record r_i. The winter snow pack then is accounted for in the estimation of effective precipitation u_i. This redistribution is estimated by

$$p_m = f\,\frac{\sum_{i=1}^{N_W} p_i}{N_m}, \tag{3}$$

where p_m is the amount of winter precipitation that should be added to each day of the melt period (cm), f is the fraction of winter precipitation that does not sublimate (dimensionless), N_W is the number of days of winter snow accumulation, p_i is the measured daily winter precipitation (cm), and N_m is the number of days during the melt period. A value of p_m is computed for each year's melt period and is added to r_i in Eqs. (1) and (2).

Convolution

The response in watershed outflow to effective precipitation can be described by the convolution integral

$$y(t) = \int_0^{\infty} h(t-\tau)u(\tau)\,\mathrm{d}\tau \tag{4}$$

where $y(t)$ is the outflow response function (m^3/s); $u(\tau)$ is effective precipitation calculated by Eq. (2) and is referred to as the signal (cm), $h(t-\tau)$ is an IRF, and $t-\tau$ represents the delay time from signal to response (Singh, 1988; Olsthoorn, 2008). If time steps of equal duration are used, the discrete form of Eq. (4) is

$$y_i = \sum_{j=0}^{i} h_{i-j}u_j \quad i = 0,1,2,\ldots,N \tag{5}$$

A single pulse of the signal u_j is dispersed within the medium in which it is transported. The function into which that single pulse is transformed is the IRF h_{i-j}, which shows the distribution of response times for modeled outflows. The response function (outflow hydrograph) is the superposition of multiple IRFs resulting from multiple pulses of the signal, where each of the IRFs is scaled by the magnitude of the corresponding pulse. The IRF is an estimate of the distribution, or spectrum, of response times to an impulse, where the mode establishes the time of peak response. The total length in time of the IRF is the system's memory, or the amount of time that the effects of an impulse remain. The IRF is assumed to be stationary in time in this application. This assumption can be assessed by calibrating the model to part of the measurement period and then executing the model for the entire period.

One of the unique advantages of convolution is that multiple flow domains can be represented by an IRF composed of multiple functions in superposition. Parametric functions useful for this