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A numerical solution for non-Darcian flow to a well in a confined aquifer using the power law function

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SUMMARY

In this study, we have obtained numerical solutions for non-Darcian flow to a well with the finite difference method on the basis of the Izbash equation, which states that the hydraulic gradient is a power function of the specific discharge. The comparisons between the numerical solutions and the Boltzmann solutions and linearization solutions have also been done in this study. The results indicated that the linearization solutions for both the infinitesimal-diameter well and the finite-diameter well agree very well with the numerical solution at late times, while the linearization method underestimates the dimensionless drawdown at early and moderate times. The Boltzmann method works well as an approximate analytical solution for the infinitesimal-diameter well. Significant differences have been found between the Boltzmann solution for a finite-diameter well and the numerical solution during the entire pumping period. The analysis of the numerical solution implies that all the type curves inside the well for different dimensionless non-Darcian conductivity k_D values approach the same asymptotic value at early times, while a larger k_D leads to a smaller drawdown inside the well at late times. A larger k_D results in a larger drawdown in the aquifer at early times and a smaller drawdown in the aquifer at late times. Flow approaches steady-state earlier when k_D is larger.

Introduction

Darcy's law has been used to simulate groundwater flow for more than 100 years. However, when the groundwater flow velocity becomes sufficiently high or sufficiently low, flow can be non-Darcian (e.g. Polubariava-Kochina, 1962; Wright, 1964; Basak and Madhav, 1979; Boast and Baveye, 1989; Venkataraman and Rao, 2000; Moutsopoulos and Tsihrintzis, 2005; Chen et al., 2003; Kohl et al., 1997; Qian et al., 2005, 2007). There are two general types of non-Darcian flow, i.e. pre-linear flow and post-linear flow. The pre-linear flow often occurs at very low Reynolds numbers (Firdaouss et al., 1997) such as in clay-rich aquitards (Teh and Nie, 2002) and in some petroleum reservoirs (Wattenbarger and Ramey, 1969), while the post-linear flow often occurs at very high Reynolds number (Zeng and Grigg, 2006) such as near the pumping wells (Sen, 1987, 1988a,b, 1989, 1990; Wu, 2002a,b; Wen et al., 2006, 2008a,b,c). In this paper, we only consider the post-linear flow for the high velocities near the pumping wells.

A key issue in non-Darcian flow is to quantify the relationship between the specific discharge and hydraulic gradient. Two formulae have been commonly used. The first one is the Forchheimer equation (Forchheimer, 1901), which states that the hydraulic gradient is a second-order polynomial function of the specific discharge. It should be pointed out that there are some alternative ways to present the Forchheimer equation (e.g. Thiruvengadam and Kumar, 1997; Nield, 2002; Moutsopoulos, 2007). For instance, Nield (2002) stated that a local time derivative inertial term and an advective inertial term should be added with the hydraulic gradient on the left hand side of the equation, while the so-called "Brinkman viscous term" should be added with the specific discharge on the right hand side of the equation. Meanwhile, Moutsopoulos (2007) pointed out that these extra terms are non-negligible only for very short times. The second one is the Izbash equation (Izbash, 1931), which states that the hydraulic gradient is a power function of the specific discharge. Many experimental data indicated that both these two functions can describe non-Darcian flow very well (Bordier and Zimmer, 2000; Yamada et al., 2005).

Up to now, many analytical solutions for non-Darcian flow have been presented. For instance, Sen (1987, 1988a,b, 1989, 1990) have obtained analytical solutions for non-Darcian flow to a well using the Boltzmann transform method, a special form of the so-called

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Nomenclature

r	distance from the center of well [L]	Q	pumping discharge [L^3/T]
r_w	effective radius of well screen [L]	$\Gamma(x)$	Gamma function
r_c	radius of well casing [L]	$K_v(x)$	the second kind v-order modified Bessel function
t	pumping time [T]	r_D	dimensionless distance defined in Table 1
$q(r, t)$	specific discharge [L/T]	r_{wD}	dimension radius of well screen defined in Table 1
$s(r, t)$	drawdown [L]	r_{cD}	dimensionless radius of well casing defined in Table 1
$S_w(t)$	drawdown inside well [L]	t_D	dimensionless time defined in Table 1
m	aquifer thickness [L]	q_D	dimensionless specific discharge defined in Table 1
S	storage coefficient of aquifer	s_D	dimensionless drawdown defined in Table 1
n	power index, an empirical constant in the Izbash equation	s_{wD}	dimensionless drawdown inside the well defined in Table 1
k	quasi hydraulic conductivity, an empirical constant in the Izbash equation	k_D	dimensionless non-Darcian flow hydraulic conductivity defined in Table 1
p	Laplace variable		

similarity method. Wen et al. (2006, 2008c) have also used the Boltzmann transform to solve the non-Darcian flow problem to a well in confined aquifers. Meanwhile, Wen et al. (2008c) pointed out that the Boltzmann transform can only be used in an approximate rather than a rigorous mathematical sense. Additionally, Camacho and Vasquez (1992) pointed out that the Boltzmann transform cannot be employed to solve non-Darcian flow problems because of non-linearity of the governing equations. Recently, Wen et al. (2008a,b) have used a linearization method coupling with the Laplace transform to solve the non-Darcian flow toward a well in a confined aquifer. They stated that this linearization procedure might work very well at late times, while at early times it will bring about some errors.

As summarized before, both the Boltzmann transform and the linearization method have some limitations. The primary limitation of the Boltzmann transform is that such a transform requires both the governing equation and the initial and boundary conditions to be transformable by the Boltzmann variable which is the ratio of the radial distance square over time. Such a requirement is often not satisfied for non-Darcian flow. The limitation of the linearization method is that it involves a quasi steady-state approximation which does not work well at early times. Fortunately, many numerical solutions have been developed for non-Darcian flow. Mathias et al. (2008), Choi et al. (1997), Wu (2002a,b), and Ewing and Lin (2001) developed numerical solutions on the basis of the finite difference scheme, whereas Ewing et al. (1999) solved the Forchheimer non-Darcian flow based on the finite element method. Kolditz (2001) has used the finite element scheme to solve non-Darcian flow in fractured rock based on the assumption that the relationship between the specific discharge and hydraulic gradient can be described by a power function. Mathias et al. (2008) developed a numerical solution for the Forchheimer non-Darcian flow to a well and compared their results with those obtained by the Boltzmann transform and linearization methods. They found that both the "Boltzmann solution" (Sen, 1988b) and the linearization solution (Wen et al., 2008a) worked well at late times, while some differences have been found at early and moderate times. Ewing et al. (1999) developed several numerical schemes, e.g. the cell-centered finite difference, the Galerkin finite element, and the mixed finite element models for the Forchheimer non-Darcian flow. So far, most of the numerical solutions for non-Darcian flow are based on the Forchheimer equation (e.g. Mathias et al., 2008; Choi et al., 1997; Wu, 2002b). However, it is equivalently important to study the Izbash non-Darcian flow with the numerical simulations.

The objectives of this paper are to develop a finite difference solution for the non-Darcian flow toward a finite-diameter well in a confined aquifer with the Izbash equation, and to verify the previous solutions obtained by the Boltzmann transform based method (Sen, 1989; Wen et al., 2008c) and the linearization approximation method (Wen et al., 2008a) using the numerical solution.

Continuity equation

Governing equation

The schematic system discussed here is the same as Papadopulos and Cooper (1967). As shown in Fig. 1, flow to a finite-diameter well which fully penetrates a confined aquifer is considered. The aquifer is assumed to be homogenous and horizontally isotropic. The whole system is hydrostatic before the start of pumping, and the pumping rate is supposed to be constant. Under these assumptions, the problem discussed here can be described as (Papadopulos and Cooper, 1967; Wen et al., 2008a,c):

$$\frac{1}{r}\frac{\partial}{\partial r}[rq(r,t)] = \frac{S}{m}\frac{\partial s(r,t)}{\partial t}, \tag{1}$$

$$s(r,0) = 0, \tag{2}$$

$$s(\infty,t) = 0, \tag{3}$$

$$2\pi r_w m q(r,t)|_{r\to r_w} - \pi r_c^2 \frac{ds_w(t)}{dt} = -Q, \tag{4}$$

in which $q(r, t)$ is the specific discharge at radial distance r and time t, $s(r, t)$ is the drawdown, S is the storage coefficient of aquifer, m is the thickness of aquifer, r_w is the effective radius of the well, r_c is the radius of well casing. In most cases, the radius of well casing r_c is larger than, instead of equal to, the effective radius of the well r_w.

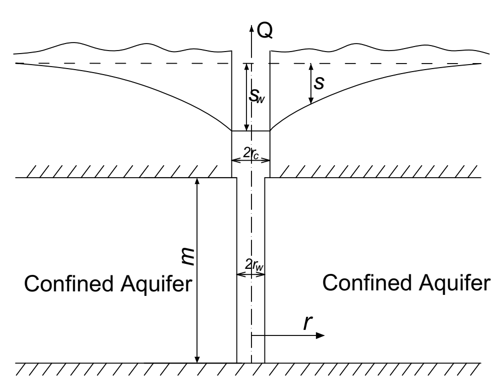

Figure 1. The schematic diagram of the problem.