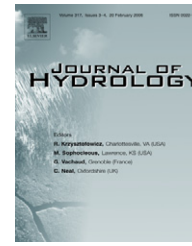
Journal of Hydrology (2008) **357**, 174–187

available at www.sciencedirect.com

ScienceDirect

journal homepage: www.elsevier.com/locate/jhydrol



Modeling of annual oscillations and 1/*f*-noise of daily river discharges

B.M. Dolgonosov *, K.A. Korchagin, N.V. Kirpichnikova

Water Problems Institute, Russian Academy of Sciences, 3 Gubkina, Moscow 119333, Russia

Received 1 August 2007; received in revised form 5 April 2008; accepted 20 April 2008

KEYWORDS
Runoff dynamics;
Fourier transform;
Power spectra;
Scaling;
Power-law trends;
Crossover frequency

Summary A frequency dependence of river runoff power spectrum is deduced from the integrated runoff and storage dynamics, derived from the mass and momentum balance equations for the water on a river catchment. The power spectrum consists of three multipliers: a $1/f^{\beta}$ noise factor with $\beta \sim 1$ reflecting a stochastic behavior of precipitation; a Lorentzian factor $1/(f_c^2 + f^2)$ damping high frequency runoff oscillations; and the sum of spectral modes of annual and intra-annual periodicity. This spectrum has two different power-law trends, separated by the crossover frequency f_c, with the exponents β and $\beta + 2$ at low and high frequencies, respectively. The theoretical results are tested on four discharge time series (3580–6575 daily records) of the river Volga at Staritsa and three rivers of the Volga basin, catchment areas varied from 1850 to 21,100 km^2. The time series are used to calculate empirical power spectra of river discharges and to construct the theoretical power spectra by fitting model parameters to the data. The calculations indicate that the spectral exponent β is approximately equal to 1 for three rivers and to 0.67 for one river. The crossover frequency f_c corresponds to a period of 12 days for large catchment areas and $\sim$3 days for small one. The comparison shows that the model and empirical spectra by their spectral trends at low and high frequencies, crossover frequency, and positions of first several spectral modes are in satisfactory agreement. To isolate an effect of fluctuations, we consider a transformed runoff time series for one of the rivers, obtained by subtraction of the multiyear average daily hydrograph from the raw time series. The transformed time series interpreted as discharge fluctuations is used to calculate the fluctuation power spectrum. This spectrum has two different power-law trends with a spectral exponent of 0.6 at low frequencies and 2.7 at high frequencies. The difference is about 2 like for the power spectrum of raw time series, but the crossover frequency slightly moves to a value corresponding to a period of 15 days.

* Corresponding author. Tel.: +7 495 135 1504; fax: +7 495 135 5415.
E-mail address: borismd@aqua.laser.ru (B.M. Dolgonosov).

0022-1694/$ - see front matter
doi:10.1016/j.jhydrol.2008.04.023

Introduction

Spectral analysis is a convenient instrument for drawing important information from river runoff time series concerning runoff oscillations and some generalized characteristics of river catchment and precipitation. In particular, spectral analysis yields the least biased results and the lowest variance in its estimates of the fractal dimension (Schepers et al., 1992).

It has been revealed in a series of works (Tessier et al., 1996; Pelletier and Turcotte, 1997; Pandey et al., 1998; Neal and Kirchner, 2000; Dahlstedt and Jensen, 2003; Feng et al., 2004; Koscielny-Bunde et al., 2006) that runoff power spectra include discrete components on the background of fluctuations in the form of $1/f$-noise. Correlation functions of these fluctuations have a power-law dependence on time lag (Beran, 1994) that is typical for fractional Gaussian noises introduced by Mandelbrot and Wallis (1968). The power-law dependence suggests long-range persistence of correlations. This contrasts with autoregressive models which have correlation functions exponentially decaying with time lag. A little earlier Hurst et al. (1965) have found that a variety of climatological and hydrological time series produce a power-law rescaled-range plot with an average exponent of $H = 0.73$. Bras and Rodriguez-Iturbe (1985) have shown that a fractional Gaussian noise with a power-law exponent of $-1/2$ yields a power-law rescaled-range plot with exponent $H = 0.75$, close to the value observed by Hurst et al. (1965).

Pelletier and Turcotte (1997) calculated power spectra of time-series data for river discharges (636 monthly records) and precipitation (49 annual records) averaged over hundreds of stations worldwide. The average power spectrum S of river discharges has a power-law dependence on frequency $S(f) \sim f^{-1/2}$ for time scales from 1 month to 10 years. However, for the precipitation power spectrum there is no identifiable persistence at time scales less than 10 years. The presence of long memory for river discharges as implied by the power-law trend of the spectrum has a significant effect on the occurrence of extreme events compared to standard autoregressive models with short memory.

It is known (Whittle, 1962) that some stochastic partial differential equations yield solutions with long-range persistence in space and time. Random processes with long-range correlations and $1/f$-noises in power spectra have probability distributions with heavy tails (Vladimirov et al., 2000). As shown by Naidenov (2004), Dolgonosov and Korchagin (2005, 2007), and Dolgonosov et al. (2006), this sort of probability distributions for river discharges and water quality indices can be deduced from some stochastic ordinary differential equations. Long-term persistence was directly observed in the decay of organic matter in water (Dolgonosov and Gubernatorova, 2005, 2007).

Feng et al. (2004) considered 17-year weekly data series of the river Hafren at Plynlimon, Wales, and showed how spectral analysis of long-term hydrological and hydrochemical data can be used to infer the travel-time distribution of water through catchments, and to measure the chemical retardation of reactive solutes at the catchment scale. They demonstrated that high frequency sampling (e.g., daily or more frequent) is particularly useful for revealing the short-term chemical dynamics that most clearly reflect the interplay of subsurface chemical and hydrological processes. The authors revealed that fluctuations of travel times are more strongly damped in the stream compared to precipitation. This was also noted by Neal and Kirchner (2000). The chemical power spectrum scales as a white noise in rainfall and as a $1/\phi$-noise in streamwater.

Baldwin and Lall (1999) investigated the seasonality in streamflows. They studied a 123-year record of daily flows and presented evidence for changes in the timing and amplitude of seasons.

Based on the spectral analysis of annual river flow, rainfall, temperature, and tree-ring data in the midwestern United States (sample sizes: river flow, 62–119 annual records for each of 26 stations; rainfall and temperature, 72–114 records, 36 stations; tree-ring, 211–513 records, 22 locations) and using the multi-taper method of spectral analysis, Rao and Hamed (2003) observed perennial oscillations in all the data having the common periodicities of 2.5–2.6, 3–3.5, 5–6, and 10.7–11.1 years. It was also found that there are periodicities of 17.9–19.6 and 60–69 years in temperature data, 21–26 years in river flow data, and 32–34 years in rainfall data. The authors noted that there is considerable evidence for drifts in these periodic components.

Pekarova et al. (2003) used the interannual spectral analysis to test selected large rivers in the world for wet and dry periods. They used annual discharge series with time scales from several decades to nearly two hundred years and demonstrated the existence of long-term discharge fluctuations (20–30 years) and a shift in the occurrence of long-term runoff extremes over the earth.

Koscielny-Bunde et al. (2006) studied temporal correlations and multifractal properties of river discharge records from 41 hydrological stations around the globe. They considered daily records of 51–171 years duration. To detect long-term correlations and multifractal behavior in the presence of trends, the authors applied the detrended fluctuation analysis as well as its multifractal realization and wavelet analysis. They found that above a crossover time of several weeks, daily runoffs are long-term correlated with the correlation function decaying as $C(\tau) \sim \tau^{-\gamma}$ with time lag τ. The exponent γ varies from river to river in a range of 0.1–0.9. The power-law decay of $C(\tau)$ corresponds to a power-law increase of the related fluctuation function $F_2(\tau) \sim \tau^{H}$ with the Hurst exponent $H = 1 - \gamma/2 = (1 + \beta)/2$, where β is the exponent in the scaling of the runoff power spectrum with frequency $S(f) \sim f^{-\beta}$.

Tessier et al. (1996) studied power spectra of thirty French rivers having areas between 40 and 200 km^2, based on daily river runoff time series from 1 day to 30 years. They showed multifractal nature of river runoffs with two frequency regions having a crossover at a period of about two weeks interpreted as the synoptic maximum (Koloshnikova and Monin, 1965). The ensemble averaged spectral exponent was estimated as $\beta = 1.3$ for the 1–16 days regime and $\beta = 0.52$ for the 1 month to 30 years regime of river runoff.

Pandey et al. (1998) analyzed daily streamflow data from 19 gauging stations from the continental USA. The length of