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Semi-analytical solution for flow in a leaky unconfined aquifer toward a partially penetrating pumping well

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Summary A semi-analytical solution is presented for the problem of flow in a system consisting of unconfined and confined aquifers, separated by an aquitard. The unconfined aquifer is pumped continuously at a constant rate from a well of infinitesimal radius that partially penetrates its saturated thickness. The solution is termed semi-analytical because the exact solution obtained in double Laplace–Hankel transform space is inverted numerically. The solution presented here is more general than similar solutions obtained for confined aquifer flow as we do not adopt the assumption of unidirectional flow in the confined aquifer (typically assumed to be horizontal) and the aquitard (typically assumed to be vertical). Model predicted results show significant departure from the solution that does not take into account the effect of leakage even for cases where aquitard hydraulic conductivities are two orders of magnitude smaller than those of the aquifers. The results show low sensitivity to changes in radial hydraulic conductivities for aquitards that are two or more orders of magnitude smaller than those of the aquifers, in conformity to findings of earlier workers that radial flow in aquitards may be neglected under such conditions. Hence, for cases were aquitard hydraulic conductivities are two or more orders of magnitude smaller than aquifer conductivities, the simpler models that restrict flow to the radial direction in aquifers and to the vertical direction in aquitards may be sufficient. However, the model developed here can be used to model flow in aquifer–aquitard systems where radial flow is significant in aquitards.

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Nomenclature

$K_{r,i}$	radial hydraulic conductivity of ith layer (LT^{-1})	r	radial distance from pumping well (L)
$K_{z,i}$	vertical hydraulic conductivity of ith layer (LT^{-1})	d	vertical distance from initial water table position to top of pumping well screen (L)
$S_{s,i}$	specific storage of ith layer (L^{-1})	l	vertical distance from initial water table position to bottom of pumping well screen (L)
S_Y	unconfined aquifer specific yield,		
$\alpha_{r,i}$	radial hydraulic diffusivity of ith layer ($\mathrm{L}^2\mathrm{T}^{-1}$)	s_i	drawdown in ith layer (L)
$\alpha_{z,i}$	vertical hydraulic diffusivity of ith layer ($\mathrm{L}^2\mathrm{T}^{-1}$)	Q	pumping well discharge rate ($\mathrm{L}^3\mathrm{T}^{-1}$)
b_i	vertical distance from initial water table position to bottom of ith layer (L)	p	Laplace transform parameter
		a	Hankel transform parameter
z	vertical distance from initial water table position (L)	t	time since onset of pumping (T)

Introduction

Leakage from adjacent aquitards has long been recognized to strongly impact the drawdown response of confined aquifers. The first major attempt to account for transient leakage in confined aquifer flow was made by Hantush and Jacob (1955), who presented the classical theory of leakage. To obtain their solution they assumed that the confined aquifer was bounded from below and above by aquitards of finite extent in which flow was entirely vertical and the effect of aquitard elastic storage was negligible. The assumption of negligible aquitard elastic storage leads to a steady-state aquitard flow problem that yields a linearly distributed hydraulic head in the aquitard. For confined aquifer flow, Hantush and Jacob (1955) adopted the assumption of horizontal flow. A major limitation of the classical theory of leakage is the assumption that aquitard storage has negligible effect on flow. Subsequently, Hantush (1960) presented the modified theory of leakage in which aquitard elastic storage was taken into account. As in classical leakage theory, flow in the aquitard was assumed to be vertical. Hantush (1960) presented numerical solutions only for early and late time. A more complete analytical solution for the problem confined aquifer flow with leakage was developed by Neuman and Witherspoon (1969a,b). They considered two confined aquifers, in which flow was assumed to be entirely horizontal, separated by an aquitard in which flow was assumed to be entirely vertical. To justify these assumptions, Neuman and Witherspoon (1969a) stated that ''when the permeabilities of the aquifers are two or more orders of magnitude greater than that of the aquitard, errors introduced by this assumption are usually less than 5%''. Recently, Sepulveda (2008) developed a semi-analytical solution for flow in a leaky confined aquifer that allows for radial and vertical flow in the aquifer and aquitards. Their results from synthetic experiments indicate that more accurate estimates of hydraulic parameters are obtainable in this case than in the case where flow is assumed to be strictly radial in aquifers and strictly vertical in aquitards.

The effect of leakage on flow in an unconfined aquifer was first considered by Ehlig and Halepaska (1976) in their numerical (finite difference) solution of a coupled confined–unconfined aquifer problem. They adopted the Boulton (1954) model to simulate unconfined aquifer flow and the Hantush and Jacob (1955) model to simulate leakage through the common boundary of the system; no analytical solution was developed. Others (Li, 2006) have analyzed data from multi-aquifer systems that include an unconfined aquifer using analytical solutions for confined aquifer flow with leakage. Zlotnik and Zhan (2005) developed an analytical solution for the problem of flow in a coupled unconfined aquifer–aquitard system, where the horizontal flow component in the aquitard is neglected. Zhan and Bian (2006) extended the work of Zlotnik and Zhan, 2005 and developed analytical and semi-analytical method for computing the leakage rate and volume induce by pumping based on the works of Hantush and Jacob (1955) and Butler and Tsou (2003). Zhan and Bian (2006) also neglect horizontal flow in the aquitard. The assumption of strictly vertical flow in the aquitard is based on the work of Neuman and Witherspoon (1969a) as discussed above. Additionally, Zlotnik and Zhan (2005) and Zhan and Bian (2006) restrict their solutions to the case of an aquitard of semi-infinite vertical extent. In this work, we develop a more general solution with respect to permissible values of aquitard hydraulic conductivity and aquitard thickness.

Malama et al. (2007) recently developed a semi-analytical solution for flow to a pumping well that fully penetrates the saturated thickness of a leaky unconfined aquifer underlain by an aquitard. The purpose of this work is to extend the semi-analytical solution of Malama et al. (2007) to a three-layered system consisting of an unconfined aquifer, an aquitard, and a confined aquifer, all of which are of infinite radial extent. The unconfined aquifer is pumped continuously at a constant rate from a well of infinitesimal radius that partially penetrates its saturated thickness. The solution is termed semi-analytical in the sense that exact an analytical solution is obtained in the double Laplace–Hankel transform space, then inverted numerically. Water release due to water table decline is simulated in the manner of Neuman (1972). Flows in the three layers are coupled by imposing drawdown and vertical flux continuity conditions at the unconfined aquifer–aquitard and confined aquifer–aquitard boundaries. In this solution we do not neglect horizontal flow in the aquitard or vertical flow in the confined aquifer, as is usually assumed in leakage theories for confined aquifers (e.g. the multi-layer approach outlined in Bruggeman (1999, p. 637) or given by Lenoach et al. (2004) in the petroleum literature). In addition, all the layers are allowed to be anisotropic. In simulating water release due to water table decline in the manner of Neuman (1972) we assume that flow in the unsaturated zone above