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Effect of variable fractal dimension on the floc size distribution of suspended cohesive sediment

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Summary Flocculation of suspended cohesive sediment, well-known to impact the floc size distribution and vertical fluxes, and cause morphodynamic changes of marine and riverine environments, is modelled by means of a population balance equation that implements a novel description of floc geometry: the capacity dimension of fractal flocs, normally assumed constant over the population, has recently been argued to change during flocculation. Our experiments have shown that a power-law function of the dimensionless floc size can conveniently be used to describe these changes. This description of floc capacity dimension is used to explore in detail the extent to which the geometrical properties of flocs affect aggregation and breakup processes, and contribute to shaping their size distribution. A comparison of experimental floc size distributions from settling column test with computed distributions for two hypotheses of floc capacity dimension (i.e., constant and variable) and two hypotheses of flocculation reactions (i.e., semi-stochastic and deterministic) are shown. This suggests that the use of variable rather than constant floc capacity dimension, and the use of semi-stochastic and asymmetric reactions rather than deterministic and symmetric, result in better predictions of the floc size distribution in the environmental conditions herein analysed.

Introduction

Flocculation of cohesive sediment suspended in natural waters is responsible for the small-scale processes of floc genesis and modifications of the floc size distribution. Flocculation also contributes to the meso- and large-scale morphological changes of estuarine environments, coastal zones, canals, rivers and water basins through sediment transport and deposition, which are related to the vertical fluxes of sediment, hence to the floc size and settling velocity distributions. The distributions of size and settling velocity in natural conditions are regulated by many climatological, hydrogeological, biochemical and physical

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List of symbols

c [ML^{-3}]	sediment concentration	L [L]	floc size
d_0	capacity dimension	L_p [L]	primary particle size
e	error function	$\alpha_{i,j}$	collision efficiency
k	primary particle number	$\gamma_{i,j}$	breakup distribution function
ℓ	dimensionless floc size	δ	primary particle capacity dimension
p	probability density function	ξ	rate of change of capacity dimension
t [T]	time	η_k [L]	floc primary particle concentration number
A [L^{-3}]	aggregation parameter	η_T [L]	total primary particle concentration number
B_i [L^{-3}]	breakup frequency	μ [ML^{-1}T^{-1}]	water dynamic viscosity
E	breakup parameter	ρ_s [ML^{-3}]	sediment density
F_y [Pa]	floc strength	ρ_w [ML^{-3}]	water density
G [T^{-1}]	turbulent shear rate	$\Lambda_{i,j}$ [L^{3}T^{-1}]	collision frequency

processes which exert their impact on the sediment dynamics at manifold time and length scales. One important aspect to the understanding of the overall sedimentological behavior of the suspended sediment is the small-scale particle—particle interaction occurring during flocculation. This is characterised by an interplay between the kinetics of aggregation and breakup reactions, and the geometric and hydraulic properties of individual flocs, which affects the floc size and settling velocity distributions and determine, consequently, the fraction of the sediment that is deposited or transported.

A numerical study of the interplay between the aggregation and breakup precesses and the geometrical properties of sedimentary flocs forming within a population requires the coupling of flocculation dynamics with a model for the geometrical properties of the flocs.

Flocculation of cohesive sediment can be modelled by means of population balance equations (PBEs) that describe the changes in number and composition of flocs within a population, and the factors that influence those changes. The first fundamental work (von Smoluchowski, 1917) focussed on the time variation of the number concentration c of a monodisperse suspension as a function of Brownian diffusion, but no kinematic processes of particle—particle interaction and floc properties were accounted for. Only after several decades, Smoluchowski's equation was extended to compute the floc number concentration in a continuum description of the floc size or volume comprising particle—particle interaction through aggregation and breakup kinetics, and processes of advection, settling, production and loss.

A major feature of these well-established PBEs is that interacting flocs are described either as Euclidean (Friedlander, 1977; McCave, 1984; O'Melia, 1980; Hunt, 1980; Krishnappan, 1990; Lee et al., 1997; Burban et al., 1989; Farley and Morel, 1986) or as fractal bodies with a constant capacity (fractal) dimension regardless of their size or growth state (Flesch et al., 1999; Serra and Casamitjana, 1998b; Zhang and Li, 2003; Kunster et al., 1997). Arguably, as suspended clay minerals are massive crystals while flocs are fluffy, porous and irregularly-shaped bodies, we expect floc geometry to undergo a transition from Euclidean to fractal during growth. While changes in perimeter-based fractal dimension and two-dimensional (2D) capacity dimension are known to appear due to, for example, changes in shear stresses, electrolyte concentration and processes of restructuring, recent works have shown evidences of changes in floc capacity dimension also at stationary environmental conditions and sediment properties, when flocs grow in time (Chakraborti et al., 2003; Maggi, 2007). These findings have raised the need to better implement the fractal description of flocs within a PBE because of a direct role of the floc fractal dimension in particle interactions during aggregation and breakup (e.g., Berka and Rice, 2005; Sato et al., 2004; Li and Logan, 2001; Winterwerp, 1998; Kunster et al., 1997; Veerapaneni and Wiesner, 1996; Burd and Jackson, 1997).

The aim of this paper is to propose the coupling of a Lagrangian population balance equation describing the time evolution of the floc size distribution with a modified model describing statistically self-similar floc geometry. Our working hypotheses are that the floc size does not scale with its mass with an invariant scaling law, i.e., a constant capacity dimension d_0, but that a variant scaling law holds during floc growth, and that the small-scale interplay between the processes of particle aggregation and breakup and geometrical properties of flocs determines the large-scale flocculation dynamics, and the resulting steady state floc size distribution. By means of the mentioned coupling, we will explore the effect of changes in 3D capacity dimension d_0 on the aggregation and breakup kinetics during flocculation, and its impact on the overall floc size distribution.

Our experimental observations suggest that the 3D floc capacity dimension d_0 decreases as a power-law of the dimensionless floc size ℓ. As fractality uniquely relates the size ℓ of a floc to the number of forming primary particles, the structure of the PBE has been elaborated in such a way as to model the suspended matter using the number concentration η_k of flocs formed by k primary particles (i.e., by a mass-based or floc primary-particle number concentration) rather than by the number concentration of flocs with sizes in a given interval. The PBE presented in this work, continuous in time but discrete in η_k, takes into account only processes of aggregation and breakup for reasons of mathematical simplicity, but advection, diffusion, settling, sediment production and loss can straightforwardly be included. The model, calibrated and validated against data obtained from settling column experiments, is analysed for different environmental parameters, and its sensitivity