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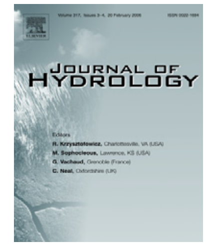

Response of sloping unconfined aquifer to stage changes in adjacent stream. I. Theoretical analysis and derivation of system response functions

Evangelos Akylas, Antonis D. Koussis *

Institute for Environmental Research and Sustainable Development, National Observatory of Athens, I. Metaxa and Vas. Pavlou, GR-15236 Palea Penteli, Athens, Greece

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Summary We study the interaction of a stream with a sloping unconfined aquifer that the stream is assumed to fully penetrate. The analysis applies to flow in a vertical section, considers the existence of a low-conductivity streambed layer and the flow in the aquifer to be induced by variations of the stream stage. Invoking the Dupuit assumption yields the 1-D Boussinesq equation, extended for a sloping base. The Boussinesq equation is linearised, the derived flow model is critiqued and an objective procedure for determining the linearisation level is developed. We solve the linear governing equation by the method of Laplace transform, with analytical inversion; the horizontal-aquifer case is treated in the zero-slope limit. The system response function is derived for the general case (sloping aquifer, sediment bed layer) and for several specific cases, and solutions are verified against known analytical results. Responses are contrasted for aquifers on positive, negative and zero slopes to step changes in the stage of streams with and without a sediment bed layer. The solutions give the aquifer stage and flow rate, the flow exchange rate at the stream–aquifer interface and the exchanged water volumes (bank storage/release).

Introduction

The interaction of a stream with an adjacent aquifer interests hydro-scientists/engineers because it occurs in a variety of cases, such as the conjunctive management of surface and ground water resources, stream base flow and the modification of a flood wave through the exchange of water across the stream banks (Sophocleous, 2002). The disparate characteristic response times of aquifer and stream flow make the computation of the modification of streamflow through its interaction with an adjacent aquifer a significant numerical challenge (Perkins and Koussis, 1996). In this work we derive the aquifer response by analyt-

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* Corresponding author. Tel.: +30 210 8109125; fax: +30 210 8103236.
E-mail address: akoussis@env.meteo.noa.gr (A.D. Koussis).

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Notation

Definition of symbols with dimension (L, length; T, time)

a	solution parameter entailing the ratio of velocity to diffusion coefficient [L^{-1}]
b	solution parameter [L^{-1}]
b_s	thickness of low conductivity sediment layer in streambed [L]
c_1, c_2	coefficients in the solution that are determined through the boundary conditions [LT]
D	diffusion coefficient [$L^2\,T^{-1}$]
D_o	diffusion coefficient, dimensionless [–]
$f(\cdots)$	function of argument in ( )
F	Laplace-transformed normalised depth of aquifer [–]
h	height of water column measured normal to the bed, depth of aquifer [L]
h_o	depth of linearisation of the aquifer [L]
H	normalised depth of aquifer [–]
H_o	normalised linearisation depth of the aquifer [–]
i	imaginary unit $(-1)^{1/2}$
K	hydraulic conductivity of aquifer [$L\,T^{-1}$]
K_s	hydraulic conductivity of sediment streambed layer [$L\,T^{-1}$]
l	streambed leakance [L]
L	length of aquifer [L]
n	drainable porosity (specific yield) of aquifer [–]
q	aquifer discharge per unit width [$L^2\,T^{-1}$]
Q	aquifer discharge per unit width normalised by KLS^2 (sloping) or KL (horizontal) [–]
R_ν	νth residual used in the development of the solution [L]
s	Laplace transform variable [T^{-1}]
s_ν	poles in the power series development of analytic functions (theorem of residua) [T^{-1}]
S	slope of aquifer basis, defined as $\sin\varphi$ [–]
t	time [T]
T	normalised time [–]
u	system response function, dimensional [T^{-1}]
U	system response function, dimensionless [–]
vol	volume of water exchanged with aquifer, per unit stream length [L^2]
V	kinematic wave linear pore velocity of aquifer, KS/n [$L\,T^{-1}$]
VOL	normalised volume of water, per unit stream length, exchanged with aquifer [–]
x	distance measured along the aquifer base [L]
X	normalised distance measured along the aquifer base [–]
y	depth at stream–aquifer interface [L]
Y	dimensionless depth at stream–aquifer interface [–]
z_ν	νth root of the equation $\tan z = f(z, a, l, L)$ [–]
η_ν	function of parameters in expression for the νth residuum, η_ν (a, l, L, z_ν, y) [–]
ζ	elevation above a datum [L]
$\xi_\nu(x)$	spatial function in expression for the νth residuum [–]
τ	dummy variable of integration over time [T]
φ	inclination angle of aquifer base against the horizontal [–]
Φ	hydraulic potential [L]
ψ	dummy variable of integration [–]
Ψ	argument of error function [–]

ical means, prescribing the variation of the stream stage as an aquifer boundary condition.

Barlow and Moench (1998) review an array of Laplace-transform-based solutions for the interaction between confined, leaky and unconfined aquifers with an adjacent stream, which the paper of Moench and Barlow (2000) summarises; they also document two computer codes for solving the interaction problem through numerical convolution. In the relevant literature the aquifer base is taken as horizontal, and, as a consequence, upon linearisation, the equation governing planar flow in an unconfined aquifers under the Dupuit approximation (Boussinesq equation) is of the pure diffusion type. In their pioneering work, Cooper and Rorabough (1963) solve that equation, deriving analytical expressions for the flow that develops in a horizontal aquifer due to a wave-like variation of the stream stage. We extend that work here to include the more realistic case of a stream interacting with an unconfined aquifer on a sloping base. Thus, after linearisation of the extended Boussinesq equation, we obtain a linear advection–diffusion-type equation that we solve by the Laplace transform method.

The structure of this first in a series of two papers is as follows. The mathematical formulation of the physics is presented first, with discussion, and also in non-dimensional form. The analytical solution methodology is detailed then, treating the case of a step change of the stream stage, from which the system response function (SRF) is derived; the solution for a horizontal aquifer is obtained in the zero-slope limit. The solutions give the aquifer stage and flow rate, the flow exchange rate at the stream-bank and the volumes exchanged (bank storage). The aquifer responses for horizontal and for positive or negative sloping bases are contrasted for step changes and for a streambed with and without a sediment layer. The added complexity of recharge is ignored mainly due to its minor influence relative to the stream.

The mathematical representation of the physics

The hydraulic description of flow in an unconfined aquifer (theory of Dupuit) of uniform hydraulic conductivity K and drainable porosity n (specific yield) approximates the pressure in the water column of height h normal to the bed as hydrostatic. The flow potential Φ is then the sum of the elevation at a point in the fluid, ζ, and the hydrostatic head