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DISCUSSION

Comment on ''Hydrological forecasting uncertainty assessment: Incoherence of the GLUE methodology'' by Pietro Mantovan and Ezio Todini

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Summary This comment is a response to the criticisms of the GLUE methodology by [Mantovan, P., Todini, E., 2006. Hydrological forecasting uncertainty assessment: Incoherence of the GLUE methodology, J. Hydrology, 2006]. In this comment it is shown that the formal Bayesian identification of models is a special case of GLUE that can be used where the modeller is prepared to make very strong assumptions about the nature of the modelling errors. For the hypothetical study of Mantovan and Todini, exact assumptions were assumed known for the formal Bayesian identification, but were then ignored in the application of GLUE to the same data. We show that a more reasonable application of GLUE to this problem using similar prior knowledge shows that gives equally coherent results to the formal Bayes identification. In real applications, subject to input and model structural error it is suggested that the coherency condition of MT06 cannot hold at the single observation level and that the choice of a formal Bayesian likelihood function may then be incoherent. In these (more interesting) cases, GLUE can be coherent in the application of likelihood measures based on blocks of data, but different choices of measures and blocks effectively represent different beliefs about the information content of data in real applications with input and model structural errors.

In Mantovan and Todini (2006, hereafter referred to as MT06), the generalised likelihood uncertainty estimation (GLUE) methodology is criticized as being incoherent in not properly taking account of the information content of an increasing number of observations that can be used in model calibration. We would suggest that the analysis of GLUE presented in MT06 is too simplistic, on two counts. The first is that they have used prior knowledge in the appli-

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cation of their formal Bayes approach that they have then ignored in their application of GLUE. The second, that in their emphasis on the coherence of learning, they have ignored the possibility of strongly over-conditioning models in the formal Bayes approach when the likelihood function overestimates the real information content of data in the face of input and model structural errors. We will briefly address each of these in turn. A full discussion will be the subject of a paper in preparation.

Comments on the experimental design and use of prior information in MT06:

In the hypothetical experiment of MT06, the model is the simple *abc* model driven by a realization of a stochastic rainfall model generated by considering the rainfall series to be formed as the accumulation of rainfall caused by ''events''. A random ''measurement error'' is added to the true output of running the rainfall realisation through the *abc* model without error. This error has zero bias, asymptotic variance of 22 units and constant lag 1 correlation of 0.8. The exact likelihood function is assumed to be known in the Bayesian identification process. A uniform prior distribution is assumed for each of the parameters *a* and *b*, while parameter *c* is assumed fixed as in MT06.

The only common assumption used by MT06 in applying the GLUE methodology is the uniform prior distributions for the *a* and *b* parameters. The informal likelihood measure assumed is the Nash-Sutcliffe efficiency measure, widely used as a performance measure in hydrological modelling. This is applied, as increasing number of observations become available, by calculating a single efficiency value over all the available data which, given the nature of the generated errors in this constructed example, means that the value does not change much as more data are added. MT06 then show that the formal Bayesian identification leads to good unbiased estimates of the parameter values, with a posterior distribution that becomes better defined as more data are added (they appear to assume that they have *perfect* knowledge of the error model parameters in doing this parameter estimation). In their application of the GLUE methodology, however, the parameter distributions are not well defined, nor do the posterior distributions become better defined as more data are added. MT06 state that their aim is not to demonstrate that using the exact likelihood in Bayesian inference leads to consistent parameter estimates but to show that GLUE, using a widely used informal likelihood, is not coherent in this sense.

We would agree that where the exact likelihood function is known *a priori* this is a well known result but let us analyse what has been done here. MT06 specify a hypothetical example in which the model is known to be correct, the structure and parameters of the error model are known precisely so that the exact likelihood function is known, and the specified prior uniform distribution of the parameters spans the true values. All this prior information is used in the Bayesian identification. It is an extreme example of the ideal case discussed in Beven (2006) (even the error model parameters are assumed to be known precisely rather than being estimated from a sample of model errors!). And then, without further comment, nearly all of this prior information is ignored when the GLUE methodology is applied to the same data set. Most importantly, the prior information about the structure of the errors is ignored in applying the Nash-Sutcliffe efficiency as an informal likelihood measure. In addition, whereas in the Bayesian inference the posterior distributions are updated as new observations are made available, in the MT06 application of GLUE the data have always been treated as a single block (such that with similar error characteristics in the hypothetical data series, there is little change in the posterior as more data are made available).

Different assumptions are therefore being made in applying the two techniques. One set of assumptions is exactly consistent with the characteristics of the data assumed known *a priori*; the other is not, with prior knowledge being neglected. It is therefore not really a great surprise that one approach gets more satisfactory results than the other!!! That is exactly what prior understanding of the problem would suggest – and, in fact, it can be easily demonstrated that if the same prior knowledge and likelihood function are used in GLUE, then very similar results will be obtained to the formal Bayes method. The use of a formal Bayes likelihood function is essentially a special case of GLUE where the user is prepared to make strong assumptions about the nature of the errors. In their own paper, MT06 cite two studies as examples of where formal likelihoods have been used in hydrological applications (Romanowicz et al., 1994, 1996), but do not actually say that in both these cases they were used in applications of the GLUE methodology.

They also overlook the fact that if those strong assumptions are not correct, then the formal Bayes method will over-condition the parameter estimates. As a simple demonstration of this, we have carried out some minor modifications to the MT06 experiment. The example is still unrealistic in that we have maintained the same model and the same input data as perfectly known. The only changes that have been made are to allow that the error model parameters are not assumed known a priori, and (unknown to the modeller) the errors are a multiplicative AR(3) rather than additive AR(1). The results are shown in Fig. 1. Applying the formal Bayes methodology of MT06 to this case, results in well defined but biased estimates of the parameters and 95% prediction limits that go below zero. Applying GLUE in the same way as in MT06 results in less well defined estimates of the parameters but where the true parameters still have high likelihood. Although GLUE does not provide probabilistic limits in the same way as the formal Bayes case, in this case (with the correct hydrological model structure) it provides prediction limits that bracket the observations, are always above zero, and have a more physically reasonable shape than the MT06 estimates. The deficiencies of the additive AR(1) model could be revealed, of course, by a post-hoc analysis of the residuals, but given the number of examples already in the literature where invalid formal likelihood functions have been used, it does not seem unreasonable to point out how making such strong assumptions can lead to misleading results using formal methods if those assumptions are not met. Further details of this and other modifications to the MT06 experiments