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Bivariate rainfall frequency distributions using Archimedean copulas

L. Zhang, Vijay P. Singh *

Department of Biological & Agricultural Engineering, Texas A & M University, 2117 TAMU, College Station, Texas 77843-2117, USA

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Summary Joint distributions of rainfall intensity and depth, rainfall intensity and duration, or rainfall depth and duration are important in hydrologic design and floodplain management. Multivariate rainfall frequency distributions have usually been derived using one of three fundamental assumptions: (1) Either rainfall variables (e.g., intensity, depth, and duration) have each the same type of the marginal probability distribution, (2) the variables have been assumed to have joint normal distribution or have been transformed and assumed to have joint normal distribution, or (3) they have been assumed independent-a trivial case. In reality, however, rainfall variables are dependent, do not follow, in general, the normal distribution, and do not have the same type of marginal distributions. This study aims at deriving bivariate rainfall frequency distributions using the copula method in which four Archimedean copulas were examined and compared. The advantage of the copula method is that no assumption is needed for the rainfall variables to be independent or normal or have the same type of marginal distributions. The bivariate distributions are then employed to determine joint and conditional return periods, and are tested using rainfall data from the Amite River basin in Louisiana, United States.

Introduction

Many water resources projects require joint probability distributions of rainfall variables (i.e., rainfall intensity, depth, and duration) which may or may not be correlated. Cordova and Rodriguez-Iturbe (1985) found that the correlation structure of rainfall intensity and duration had a significant effect on surface runoff. Hashino (1985) generalized the Freund bivariate exponential distribution (Freund, 1961) to represent the joint probability distribution of rainfall intensity and maximum storm surge in Osaka Bay, Japan. Singh and Singh (1991) derived a bivariate probability density function with exponential marginals to describe the joint distribution of rainfall intensity and depth. Representing rainfall occurrence by a Poisson model, Bacchi et al. (1994) derived bivariate distributions with marginal exponential distributions for rainfall intensity and duration (Long and Krzysztofowicz, 1992, 1995).

* Corresponding author.
E-mail address: vsingh@tamu.edu (V.P. Singh).

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Notations

$C_{U_1|U_2=u_2}(u_1)$ conditional joint copula (probability) function given $U_2 = u_2$
$C_{U_1|U_2\leqslant u_2}(u_1)$ conditional joint copula (probability) function given $U_2 \leqslant u_2$
$C(F_{X_1}(x_1),\ldots,F_{X_n}(x_n))$ multivariate copula function of n dependent variables
D($\cdot$) Debye function
$F_X(x)$, $F_Y(y)$ cumulative probability function of variable X and Y
$f_X(x)$, $f_Y(y)$: probability density function of variable X and Y
$H(x,y)$ joint cumulative probability function of two dependent variables X and Y
$h(x,y)$ joint probability density function of two dependent variables x and y
$H_{X|Y}(x|y)$ conditional joint cumulative probability of $X \geqslant x$ given $Y = y$
$H'_{X|Y}(x|y)$ conditional joint cumulative probability of $X \geqslant x$ given $Y \leqslant y$
$H(x_1, x_n, x_3, \ldots, x_n)$ multivariate distribution function of n dependent variables
$T_{X,Y}(x,y)$ joint return period represents either x or y or both x and y values are exceeded
$T_X(x)$ return period of variable x
$T_Y(y)$ return period of variable y
$T_{X|Y}(x|y)$ conditional joint return period of $X \geqslant x$ given $Y = y$
R, V, D represents: rainfall intensity, depth and duration, respectively
u_i uniform distributed random variables on $[0,1]$
x_c computed value
x_0 observed value
θ copula parameter
ϕ copula generator
τ Kendall's tau correlation coefficient

For Gamma distribution
α, β parameters of gamma distribution

For Weibull distribution
γ, β parameters of Weibull distribution.

For exponential distribution
λ parameter of exponential distribution.

For lognormal distribution
μ, σ parameters of lognormal distribution.

Overcoming the shortcomings of a previously derived flood frequency distribution (DFFD) model in which rainfall intensity and duration were considered independent of each other, Kurothe et al. (1997) improved the DFFD model for negatively correlated rainfall intensity and duration. The results obtained on the Davidson watershed in North Carolina showed that the negative correlation structure of rainfall intensity and duration markedly influenced the estimated flood quantiles. Goel et al. (2000) extended their work on the DFFD model by including both the positive and negative correlation structures of rainfall intensity and duration. Their results from four Indian watersheds and 1 US watershed showed the importance of both positive and negative correlation structure on the performance of the DFFD model. Yue (2000a) applied a bivariate normal distribution to represent the joint distribution of peak rainfall intensity and depth which were correlated with each other; the Box–Cox transformation was used to normalize the original marginals. Yue (2000b,c, 1999) applied bivariate lognormal, Gumbel mixed and Gumbel logistic distributions for multivariate rainfall frequency analysis.

In above studies, one of two fundamental assumptions has been made. Either rainfall variables each have the same type of marginal probability distribution or the variables have been assumed to have the normal distribution or have been transformed to have the normal distribution. In practice, however, rainfall variables may have different distributions. Relaxing these assumptions, Long and Krzysztofowicz (1992) studied the Farlie–Gumbel–Morgenstern and Farlie polynomial bivariate probability density functions which are independent of marginals. They discussed both the limitations and advantages of these two bivariate densities in multivariate hydrological frequency analysis. In another study, Long and Krzysztofowicz (1995) constructed bivariate probability density functions as $h(x,y) = f(x)g(y)[1 + \theta c(F(x), G(y))]$ where $c[F(x), G(y)]$ characterizes the covariance structure, $F(x)$is the marginal distribution of X and $G(y)$ is the marginal distribution of Y. They discussed that the covariance characteristic c can describe either positive or negative dependence up to Fréchet bounds, and the mutual regression dependence of X and Y. A major advantage is that within certain constraints the shape of the bivariate density and the degree of association between X and Y can be controlled.

Kelly and Krzysztofowicz (1995) applied the meta-Gaussian distribution in hydrology. This distribution uses the bivariate normal copula and it differs from the bivariate normal distribution in that it relaxes the normality assumption of original variates, avows for nonlinear and heteroscedastic dependence between the variates, and can represent any degree of dependence between the variates. Their work probably was among the first, considering different marginal distributions with different covariance structures in bivariate frequency analysis. Herr and Krzysztofowicz (2005) derived a generic form of a bivariate probability distribution, using the meta-Gaussian distribution, for precipitation amounts, fully characterizing the stochastic dependence between precipitation amounts at two stations. They discussed its estimation from data at length.

Salvadori and De Michele (2004) presented theoretical aspects of frequency analysis based on copulas. They also presented case studies to illustrate the power of the copula-based analysis. Favre et al. (2004) developed a methodology for modeling extreme values using copulas. Elliptic and Archimedean copulas and copulas with quadratic section were tested on peak flows from a watershed in Quebec,