

Estimation of areal reduction factors using a mixed gamma distribution

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Summary This study proposes a methodology to estimate the areal reduction factor (ARF) using mixed probability density functions. In practice, ARFs are estimated by using contemporaneous rainfall data from a high density rain gauge network over the basin, which are rarely available. The methodology proposed in this study uses all rainfall data available, whether or not they are concurrent with others. A mixed probability density function is introduced to explain both the rainfall intermittency and variability, and is applied to both point rainfall and areal average rainfall, which are compared to estimate the ARFs. In this study, a mixed gamma distribution was applied for the Geum river basin, Korea, to estimate the ARFs of a one-day (or, 24 h) rainfall as an example. The new method was easier to apply and provided similar results with those of other studies.

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Introduction

One of the general characteristics of rainfall is the reduction in the areal average rainfall with increase in the area. For this reason, we have to use the areal average rainfall, instead of point rainfall, for the analysis of runoff or in the design of hydraulic structures. However, it is not so

common to have enough number of rain gauges and their record lengths is generally not long enough to make consistent time series of areal average rainfall. Thus, the point rainfall measurements that have been recorded for a long time are used to estimate the design rainfall amount, which is then converted into the areal average rainfall amount by introducing the areal reduction factor (ARF).

Many researches can be found on the ARF (Roche, 1966; Rodriguez-Iturbe and Mejia, 1974; Omolayo, 1993; Sivapalan and Blöschl, 1998; Asquith and Famiglietti, 2000; De Michele et al., 2001). Especially, Roche (1966) developed a theoretical approach based on the correlation structure of flash flood to handle the problem of the ARF. Rodriguez-Iturbe

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and Mejia (1974) improved this approach by introducing the variance function and the variance reduction factor. This improved approach was also used by Sivapalan and Blöschl (1998) to estimate the ARF for extreme flash floods. Asquith and Famigletti (2000) also derived the ARF of three regions of Texas based on the annual-maxima centered method (similar to storm centered method), which were benefited by the high rain gauge density of Texas. The statistical approach of De Michele et al. (2001) derives the ARF by considering spatial–temporal scaling characteristics of a rainfall. Lee (1987), Lee and Lee (1987), Kim and Yoon (1992), Heo and Heo (2001) and Jeong et al. (2002) have presented results on the ARF in Korea. The Ministry of Construction and Transportation (MOCT, 2000) also published a report on ARF for several important river basins in Korea.

However, most of these methods require rather high rain gauge density and long contemporaneous measurements of rainfall for most rain gauges. Some other methods require handling a complex procedure (Rodríguez-Iturbe and Mejia, 1974) or a non-linear system (De Michele et al., 2001). Without enough available data, none of methods can be applied easily to estimate the ARF. This situation is also the same in Korea. Even in cases when the rain gauge density is high and the period of data collection is long, the ARF derivation procedure is a boring routine. The ARF estimated may also contain numerous uncertainties mainly due to the high spatial–temporal variability of a rainfall.

In this study, a new methodology for deriving the ARF is introduced. This method is based on the concept of rainfall intermittency represented by a mixed probability distribution. The mixed distribution is known to well represent the rainfall intermittency (Kedem et al., 1990), and to well distinguish the characteristics of an areal average rainfall with respect to the area size. As an example, this study applies the two parameter Gamma distribution to the daily rainfall data collected in the Geum river basin, Korea during the wet season (June–September). The ARF estimated is also compared with other research results for validation.

Mixed gamma distribution and ARF derivation

Derivation of rainfall quantiles using mixed gamma distribution

Mixed distribution

A mixed distribution indicates the combination of discrete distribution and continuous distribution. Many researchers have tried recently to characterize rainfall by using a mixed distribution after Kedem et al. (1990), who proposed the use of mixed distribution to represent rainfall. Shimizu (1993) also theoretically analyzed mixed log-normal distribution. Yoo et al. (2002) and Yoo and Jung (2002) of Korea have tried to apply mixed distribution for the optimization of a rain gauge network and for the frequency analysis of rainfall.

The theoretical background of mixed distribution is as follows. If we let the rainfall at a point be a random variable denoted by X , which will be zero for the dry period with its probability $1 - p$:

$$P(X = 0) = 1 - p \quad (1)$$

where p represents the probability of rainfall (or the wet probability). However, under the condition of rainfall (that is, during the wet period), X is continuously distributed. The probability of X which will be smaller or equal to x can be expressed by a cumulative probability density function $F(x)$:

$$P(X \leq x | X > 0) = F(x), \quad x > 0 \quad (2)$$

Thus, the distribution of rainfall considering both the wet and dry periods can be represented by a mixed distribution (Kedem et al., 1990).

$$G(x) = (1 - p)H(x) + pF(x) \quad (3)$$

where $G(x)$ is a distribution function of X including the dry period, and $H(x)$ is defined as a step function as:

$$\begin{aligned} H(x) &= 1 & \text{if } x = 0 \\ H(x) &= 0 & \text{if } x > 0 \end{aligned} \quad (4)$$

Thus, $F(x)$ or its first derivative $f(x)$ represents the continuous part of a mixed distribution, and $H(x)$ is a discrete part representing the dry period. Mean rainfall intensity $E[X]$ equals to the expectation value of $G(x)$, and under the wet condition, the mean rainfall intensity $E[X|X > 0]$ is defined as the expectation value of $F(x)$.

Mixed gamma distribution

Gamma distribution is one of the most popular distributions used to represent rainfall along with the exponential and log-normal distributions (Wilks, 1998). This study chose the Gamma distribution, because it is believed to be more flexible for various applications. The cumulative probability density function $F(x)$ is defined as:

$$F(x) = \int_0^x f(x) dx = \int_0^x \frac{1}{\alpha \Gamma(\beta)} \left(\frac{x}{\alpha}\right)^{\beta-1} e^{-x/\alpha} dx, \quad x, \alpha, \beta > 0 \quad (5)$$

where $f(x)$ is the probability density function of x following the two-parameter Gamma distribution, and $\Gamma()$ is the gamma function. The parameters α and β decide the scale and shape of the distribution, which can be estimated using the Maximum Likelihood Method (Rao and Hamed, 2000). First, the parameter is expressed as:

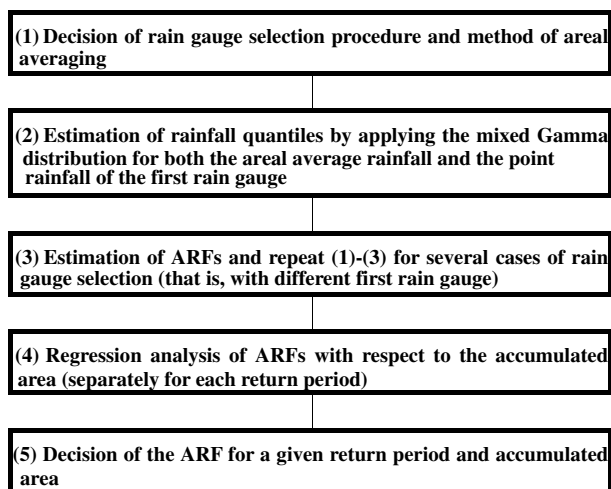


Figure 1 Procedure for estimating ARF using mixed distributions of areal average rainfall.

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