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Characteristics of unsteady flow to wells in unconfined and semi-confined aquifers

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Abstract

An approximate solution by Zhan and Zlotnik for flow to a horizontal or slanted well in an unconfined aquifer is shown to be general enough to include the earlier solutions of Theis, Hantush and Jacob, Boulton and Neuman and is used to investigate the behavior of flow to a well in unconfined and semi-confined aquifers. The semi-confined aquifer is bounded on top by one or more aquitard layers containing a free surface, which is allowed to draw down with time as flow is abstracted from the well. Drawdown curves for unconfined and semi-confined aquifers are observed to have similar shapes but greatly different time scales, drawdown magnitudes and drawdown distributions along vertical lines in the pumped aquifer. This emphasizes the importance in applications of choosing mathematical models that correctly fit aquifer geologies. The Zhan and Zlotnik solution utilizes several approximations, and comparing solutions for flow to vertical wells in unconfined and semi-confined aquifers with the exact Neuman and Boulton solutions tests the accuracy of these approximations. Then the solution is used to plot drawdown contours around a horizontal well in an unconfined aquifer, and a number of reasons are given for the relative efficiency of using a horizontal well to abstract water from an aquifer.

Keywords: Ground water; Aquifers; Unconfined aquifers; Aquitards; Drawdown

1. Introduction

Many papers have been published on the subject of unsteady flow to wells since Theis (1935) published his landmark paper on flow to a well in a confined aquifer. Significant contributions to this topic were made later by Hantush and Jacob (1955), with their solution for unsteady flow to a well in a leaky aquifer, by Boulton (1954, 1963), with his solution for delayed-yield flow to a well that was later shown by Boulton (1973); Cooley and Case (1973) to describe flow to a well in a semi-confined aquifer, and by Neuman (1974), with his solution for delayed-response flow to a well in an unconfined aquifer. More recently, Zhan and Zlotnik (2002) have shown how a solution can be calculated for flow to a horizontal or slanted well in an unconfined aquifer. This solution was obtained by using the Stehfest (1970) algorithm to invert the Laplace transform solution for flow to a point sink. Then this

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fundamental solution was distributed along a straight-line segment to model flow to a well screen with finite length. The following work will discuss the equations and some extensions of the equations that describe the solution obtained by Zhan and Zlotnik (2002). In particular, it will be shown that the Zhan–Zlotnik (2002) solution is general enough to calculate flow to a horizontal, slanted or vertical well in any of the different types of aquifers considered previously by Theis (1935); Hantush and Jacob (1955), Boulton (1954, 1963) and Neuman (1974). An empirical constant, α, in the Zhan-Zlotnik (2002) solution will be rewritten in terms of physically meaningful aquifer parameters, and it will be shown how this solution can be used to describe flow to a well when a number of overlying aquitards exist between the pumped aquifer and a free surface. Then example applications will be used to illustrate some characteristics of flow to wells in unconfined and semi-confined aquifers.

2. Flow to a point sink in an unconfined aquifer

Flow to a point sink in an unconfined aquifer is shown in Fig. 1. The sink is located at the point $(r,z)=(0,\xi)$, where r is a horizontal radial coordinate and z is a vertical coordinate that is positive in the upward direction. The impermeable bottom aquifer boundary coincides with the plane $z=0$, and the free surface coincides initially with the plane $z=B$. The aquifer, in general, is anisotropic with principal values K_H and K_V for the hydraulic conductivity tensor in the horizontal and vertical directions, respectively. The aquifer is also assumed to be homogeneous, which means that K_H and K_V must not change with either r or z. This is a particularly stringent requirement, since anisotropy often results from horizontal layering in fluvial deposits of sand and gravel, and the homogeneity requirement means that many thin layers with a cyclic repetition of hydraulic conductivity sequences must occur throughout the entire saturated thickness of the aquifer. This is the same aquifer considered by Neuman (1974) when he obtained his well-known solution for unconfined flow to a well.

Flow to a point sink in an unconfined aquifer is modelled by solving the following set of equations:

$$\frac{K_H}{r}\frac{\partial}{\partial r}\left(r\frac{\partial s}{\partial r}\right)+K_V\frac{\partial^2 s}{\partial z^2}=S_s\frac{\partial s}{\partial t} \quad (0<r<\infty, 0<z<B, 0<t<\infty) \tag{1}$$

$$\lim_{r\to 0}\left(r\frac{\partial s}{\partial r}\right)=-\frac{Q}{2\pi K_H}\delta(z-\xi) \quad (0<z<B, 0<t<\infty) \tag{2}$$

$$s(\infty,z,t)=0 \quad (0<z<B, 0<t<\infty) \tag{3}$$

$$\frac{\partial s(r,0,t)}{\partial z}=0 \quad (0<r<\infty, 0<t<\infty) \tag{4}$$

$$K_V\frac{\partial s(r,B,t)}{\partial z}+S_y\frac{\partial s(r,B,t)}{\partial t}=0 \quad (0<r<\infty, 0<t<\infty) \tag{5}$$

$$s(r,z,0)=0 \quad (0<r<\infty, 0<z<B) \tag{6}$$

where s, drawdown; r, radial coordinate; z, vertical coordinate; t, time; K_H and K_V, horizontal and vertical hydraulic conductivities, respectively; S_s, specific storage; S_y, specific yield; Q, flow rate to the sink; B, initial aquifer thickness; $(0,\xi)=(r,z)$ coordinates of the sink and $\delta(x)=$Dirac's delta function. Eq. (4) shows that the impermeable bottom boundary coincides with the plane $z=0$, and Eq. (5) is a

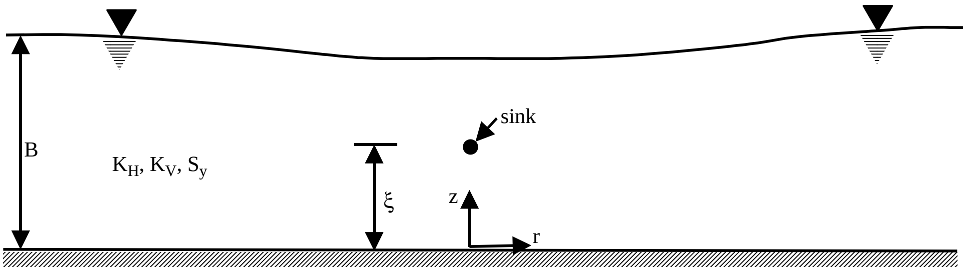

Fig. 1. Flow to a sink in an unconfined aquifer.