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Efficient nonlinear modeling of rainfall-runoff process using wavelet compression

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Summary This investigation proposes the wavelet-based efficient modeling of nonlinear rainfall-runoff processes and its application to flood forecasting in a river basin. Inspired by the theory of wavelet transforms and Kalman filters, based on the excellent capacity of the Volterra model, a time-varying nonlinear hydrological model is presented to approximate arbitrary nonlinear rainfall-runoff processes. A discrete wavelet transform (DWT) is used to decompose and compress the Volterra kernels, generating smooth reparametrizations of the Volterra kernels, reducing the number of coefficients to be estimated. Kalman filters were then utilized to online estimate compressed wavelet coefficients of the Volterra kernels and thus model the time-varying nonlinear rainfall-runoff processes. Kalman filters and the Volterra model that had been used over recent decades in the nonlinear modeling of rainfall-runoff processes, typhoon or storm events over Wu—Tu and Li—Ling watersheds are chosen as case studies were used herein to verify the suitability of a combination of wavelet transforms. The validation results indicated that the proposed approach is effective because of the multi-resolution capacity of the wavelet transform, the adaptation of the time-varying Kalman filters and the characteristics of the Volterra model. Validation results also reveal that the resulting method improves the accuracy of the estimate of runoff for small watersheds in Taiwan.

Introduction

As Taiwan is located in the major typhoon track in the western Pacific Ocean, typhoons are an influential weather phenomenon in Taiwan, where short, steep upstream channels characterize all watersheds. The associated heavy rainfall and flooding are one of the disasters which cause the greatest loss of property and life in the area. It is therefore essential to study the relationship of the rainfall and runoff processes and develop a flood forecasting system to provide protection and warning systems.

Flood forecasting is based on rainfall-runoff relationship modeling. Conventional rainfall-runoff schemes employ a linear time invariant response function to approximate the dynamic behavior of a rainfall-runoff relationship. Developed

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by Sherman in 1932, Unit hydrograph (UH) plays a critical role in predicting runoff hydrograph. However, UH has many limitations in application. For instance, linearity and time invariance form the basis of UH theory. Despite the extensive use of UH, the linear time invariant relationship between rainfall and runoff is not strictly valid.

Consequently, the response function with time-varying characteristics, i.e., the ability to adapt to its surrounding environment, is necessary. In addition, many nonlinear systems exist. Almost all practical systems, including rainfall-runoff processes, are nonlinear. The modeling of a nonlinear system must be studied. Inspired by the theory of wavelet transform and the Kalman filter, and based on the excellent power of the Volterra model, a time-varying nonlinear hydrologic model is developed for approximating arbitrary nonlinear rainfall-runoff processes herein.

Rainfall-runoff systems are generally non-linear and time-variant. Their parameters vary in time and space, and when they are assumed constant they are only so by assumption. The inadequacy of the model itself, parameter uncertainty, errors in the data used for parameter estimation and inadequate understanding of the rainfall-runoff process, owing in part to randomness, may cause errors in a rainfall-runoff model. Incorporating the Kalman filter in a rainfall-runoff model may reduce the error in the runoff prediction arising from the uncertainty caused by the physical process, the model and the input data (Lee and Singh, 1998). Lee and Singh also reviewed in detail several applications of the Kalman filter to hydrology.

Identification involves the determination of an unknown system based on input—output information in an uncertain environment. Volterra filters have been successfully used in modeling numerous physical applications, especially in signal processing and system identification. The identification of nonlinear systems depends on an accurate description of such. Describing nonlinear systems without a general mathematic model is difficult. The Volterra model, which is polynomial (Feng, 1999), can be treated as a general representation of a nonlinear system. In this paper, a nonlinear second-order Volterra model is adopted to model the nonlinear rainfall-runoff process. However, the conventional method for identifying the Volterra model is inefficient and inaccurate because too many parameters must be estimated.

Nikolaou and Mantha (2000) demonstrated how to use wavelets for the reparametrization of second-order Volterra models in terms of a substantially smaller number of coefficients. The resulting structure retains several of the advantages of the Volterra structure, while being parsimonious, thus making feasible the identification of Volterra model form experimental data. A simulation study on polymerization reactor elucidates their proposed approach. Herein in this paper the wavelet transform is similarly applied to decompose and compress the Volterra kernels, and then Kalman filter is adopted to model the nonlinear time-varying rainfall-runoff process.

Wavelets are becoming an increasingly important tool to process image and signal. Wavelets effectively extract both time and frequency-like information from a time-varying signal. Wavelets and Kalman filters can handle the non-stationary signal; many applications combine these two tools. Cheng and Sun (1996) used the theory of wavelet packet analysis to decompose the primitive measurements into two parts, namely the ''trend'' and ''fluctuation'' of the measurement. Hong et al. (1998) utilize the DWT to decompose the state variables of the Kalman filter into different components at the desired resolution level, and then process the prediction, correction and update procedure using the decomposed state variables.

In the field of hydrology, wavelets have been essentially used in order to identify coherent convective storm structures and characterize their temporal variability (Szilagyi et al., 1999) but also in order to explain the non-stationarity of karstic watersheds (Labat et al., 2000). Wavelet analysis of rainfall rates and runoffs and wavelet rainfall-runoff cross-analysis give meaningful information on the temporal variability of the rainfall-runoff relationship. Wavelets were regarded as a non-parametric data generation tool proposed by Bayazit and Ahsoy (2001) and applied to annual and monthly streamflow series taken from Turkey and USA. The idea behind their suggested method is decomposition of data into its details and later reconstruction by summation of the details randomly to generate new data. Chou and Wang (2002) applied the DWT to decompose and compress the UH. Moreover, a wavelet-based linearly constrained least mean squares (WLCLMS) algorithm is also used to estimate on-line the wavelet coefficients of the UH. Their proposed approach allows the UHs to vary in time and accurately predicts runoff from a basin, thus making it highly promising for flood forecasting.

Smith et al. (2004) define a quantitative index that describes the variability in streamflow discharge in terms of filtering of the rainfall signal for an event. They base this index on the interpretation that the wavelet coefficients in the $M \times N$ transform matrix correspond to the correlation between the wavelet and the signal. Chou and Wang (2004) proposed a multi-model method using a wavelet-based Kalman filter (WKF) bank to simultaneously estimate decomposed state variables and unknown parameters for real-time flood forecasting. Their validations showed that the approach is effective because of the decomposition of wavelet transform, the adaptation of the time-varying KF and the multi-model method properties.

This study is based on the premise that some prior knowledge about Volterra kernels can be translated into a sparseness pattern of the DWT of the model to be estimated, as proposed by Nikolaou and Mantha (2000). In this investigation, the DWT is first used to transform the original Volterra kernels. A priori information is then used to eliminate several coefficients by setting them equal to zero. The structure of the original model is retained, and the resulting parsimonious model is simpler to identify. It can be easily converted into original form by applying the inverse discrete wavelet transform (IDWT). The set of coefficients of the DWT of the Volterra kernels that are set to zero can be iteratively refined. The Kalman filter algorithm is then used to update the compressed wavelet coefficients in each time index, enabling the runoff in the time domain to be predicted.

The rest of this article is organized as follows. First, the structure of the second-order Volterra models in discrete-time systems is described, after having been identified by the conventional matrix method in the time domain. Then, the wavelet and the DWT are defined. The decomposition and compression of Volterra models using the DWT is also