Journal of Hydrology (2006) **331**, 146–160

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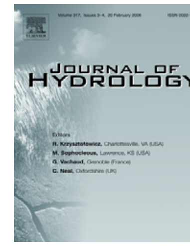

Takagi–Sugeno fuzzy inference system for modeling stage–discharge relationship

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Received 27 July 2005; received in revised form 10 May 2006; accepted 10 May 2006

KEYWORDS
Fuzzy logic;
Takagi–Sugeno fuzzy inference system;
Artificial neural network;
Hysteresis effect;
Loop rating curve;
Clustering

Summary Direct measurement of discharge in a stream is not only difficult and time consuming but also expensive. Therefore, the discharge in a stream is related to the stage through a number of carefully measured discharge values. A relationship between stages and corresponding measured discharges is usually derived using various graphical and analytical methods. As the relationship between stages and measured discharges is not linear, conventional methods based on least squares regression analysis for fitting a relationship are unable to model the non-linearity in the relationship and spatially in the cases when hysteresis is present in the data. The aim of the present study is to investigate the potential of Takagi–Sugeno (TS) fuzzy inference system for modeling stage–discharge relationships and the investigations are illustrated by application of the model to observed gauge and discharges of various gauging stations in Narmada river system, India. A step by step procedure for developing TS fuzzy model is also presented. The results show that the TS fuzzy modeling approach is superior than the conventional and artificial neural network (ANN) based approaches. Comparison of the models on hypothetical data set also reveals that the fuzzy logic based approach is also able to model the hysteresis effect (loop rating curve) more accurately than the ANN approach.

Introduction

Stream flow information is important for effective and reliable planning and management of various water resources activities and the assessment, management and control of water resources can be effective if accurate and continuous information on river-flow is available. Generally a network of river gauging stations provides continuous information on river stage and sparse information of corresponding discharges. Thus, the continuous discharge data corresponding to observed gauge can be obtained by developing a stage discharge relationship and using this relationship to convert the recorded stages into corresponding discharges. This

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0022-1694/$ - see front matter
doi:10.1016/j.jhydrol.2006.05.007

relationship is determined by correlating measurements of discharge with the corresponding observations of stage (Maidment, 1992). However, under certain conditions (flatter gradients and constricted channels) the discharge for a flood on a rising stage differs from that on the falling stage. This phenomenon is called hysteresis and results in a looped stage—discharge curve (Tawfik et al., 1997) for floods with different stage—discharge relations for rising and falling water stages. Rating curve development approaches can be categorized into three main groups: the single curve approach, the raising and falling approach, and the Jone's approach (Tawfik et al., 1997). DeGagne et al. (1996) documented the process of developing a decision support system for the analysis and use of stage—discharge rating curve while other possible models have been proposed by Gawne and Simonovic (1994) and Yu (2000).

The functional relationship between stage and discharge is complex and can not always be captured by these traditional modeling techniques (Bhattacharya and Solomatine, 2005). In the real world, stage and discharge relationship do not exhibit simple structure and are difficult to analyze and model accurately. Therefore, it seems necessary that soft computing methods e.g. artificial neural network (ANN) and fuzzy logic, which are suited to complex non-linear models, be used for the analysis. There are several applications of ANNs in stage—discharge modeling. Jain and Chalisgaonkar (2000) used three layer feed forward ANNs to establish stage—discharge relationship. Bhattacharya and Solomatine (2005) have found that the predictive accuracy of ANN model is superior than the traditional rating curves. The effectiveness of an ANN with a radial basis function was explored by Sudheer and Jain (2003). The ANN based approaches have also provided promising results in modeling loop rating curves (Jain and Chalisgaonkar, 2000; Sudheer and Jain, 2003).

The purpose of this study is to investigate and explore the potential of an alternate soft computing technique for stage discharge modeling based on fuzzy logic. The ability of fuzzy logic to model nonlinear events makes it even more important to investigate its ability to model stage discharge relationship. Uncertainty in conventional gauge—discharge rating curves involves a variety of components such as measurement noise, inadequacy of the model, insufficiency of river flow conditions, etc. Fuzzy logic based modeling approach has a significant potential to tackle the uncertainty problem in this field and to model nonlinear functions of arbitrary complexity. Other advantage of fuzzy logic is its flexibility and tolerance to imprecise data (Zadeh, 1999). Fuzzy rule based modeling is a qualitative modeling scheme where the system behavior is described using a natural language (Sugeno and Yasukawa, 1993). The transparency of the fuzzy rules provides explicit qualitative and quantitative insights into the physical behavior of the system (Coppola et al., 2002). The application of fuzzy logic as a modeling tool in the field of water resources is a relatively new concept although some studies have been carried out to a limited extent and these studies have generated considerable enthusiasm. Fuzzy rule based modeling has been attempted in water resources management, reservoir operation, flood forecasting and other areas of water resources analysis (Bardossy and Duckstein, 2002; Fontane et al., 1997; Kindler, 1992; Mujumdar and Sasikumar, 2002; Panigrahi and Mujumdar, 2000; Sasikumar and Mujumdar, 1998; Deka and Chandramouli, 2003; Lohani et al., 2005). This paper is concerned with the application of an emerging, powerful soft computing technique fuzzy logic to stage discharge rating curves.

Overview of fuzzy logic

The classical theory of crisp sets can describe only the membership or non-membership of an item to a set. While, fuzzy logic is based on the theory of fuzzy sets which relates to classes of objects with unsharp boundaries in which membership is a matter of degree. In this approach, the classical notion of binary membership in a set has been modified to include partial membership ranging between 0 and 1 (Zadeh, 1965). The membership function is described by an arbitrary curve suitable from the point of view of simplicity, convenience, speed, and efficiency. A sharp set is a sub set of a fuzzy set where the membership function can take only the values 0 and 1.

The range of the model input values, which are judged necessary for the description of the situation, can be portioned into fuzzy sets. The process of formulating the mapping from a given input to an output using fuzzy logic is called the fuzzy inference (Jang, 1993). The basic structure of any fuzzy inference system is a model that maps characteristics of input data to input membership functions, input membership function to rules, rules to a set of output characteristics, output characteristics to output membership functions, and the output membership function to a single-valued output or a decision associated with the output (Jang et al., 2002). In rulebased fuzzy systems, the relationships between variables are represented by means of fuzzy if-then rules e.g. ''*If* antecedent proposition *then* consequent proposition''. Depending on the particular structure of the consequent proposition, three main types of fuzzy models are distinguished as: (1) Linguistic (Mamdani Type) fuzzy model (Zadeh, 1973; Mamdani, 1977) (2) Fuzzy relational model (Pedrycz, 1984; Yi and Chung, 1993) (3) Takagi—Sugeno (TS) fuzzy model (Takagi and Sugeno, 1985). A major distinction can be made between the linguistic model, which has fuzzy sets in both antecedents and consequents of the rules, and the TS model, where the consequents are (crisp) functions of the input variables. Fuzzy relational models can be regarded as an extension of linguistic models, which allow for different degrees of association between the antecedent and the consequent linguistic terms. In this work, the TS fuzzy model is employed to develop stage discharge rating curve. These models are relatively easy to identify and their structure can be readily analyzed (Lohani et al., 2005).

Takagi—Sugeno fuzzy inference system

A fuzzy rule-based model suitable for the approximation of many systems and functions is the Takagi—Sugeno (TS) fuzzy model (Takagi and Sugeno, 1985). In the TS fuzzy model, the rule consequents are usually taken to be either crisp numbers or linear functions of the inputs

$$R_i: \text{ IF } x \text{ is } A_i \text{ THEN } y_i = a_i^T x + b_i, \quad i = 1, 2, \ldots M \tag{1}$$

where $x \in \Re^n$ is the antecedent and $y_i \in \Re$ is the consequent of the ith rule. In the consequent, a_i is the parameter