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Nonlinear transformation of unit hydrograph

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Summary Unit hydrograph (UH) and its numerous derivatives have been popular for estimation of flood hydrographs. Two major assumptions still overshadow UH applications. One is the linearity and the other is time invariance. In theory, only peak discharge of an equilibrium hydrograph follows linear proportionality to excess rainfall intensity. In trying to relax the linearity constraint, this paper aims to propose a nonlinear way of transforming a given UH to other general hydrographs. The transformation or mapping technique relies on a simple rainfall ratio raised to a power less than unity. The case of nonlinear transformation is illustrated for a number of watershed geometries with either known kinematic wave analytic solutions or observed data. The nonlinear UH approach also relaxes the assumption of constant time base of the UH. The proposed nonlinear UH transformation may thus be viewed as a major step in closing the gap between physically based and traditional UH-based surface runoff simulation approaches.

Introduction

Unit hydrograph (UH) is perhaps the most widely practiced rainfall-runoff transformation technique known to surface water hydrologists. UH and its many varieties continue to be the favorite in flood hydrograph estimation studies. UH has steadily been a tempting research subject as well, as witnessed by numerous papers in the literature which have dealt with UH issues.

The underlying characteristics of unit hydrograph theory, as proposed by Sherman (1932), are:

1. UH is a lumped model of transforming (excess) rainfall to (direct) runoff. This single transformation model normally takes a spatially averaged excess rainfall and converts that into a hydrograph. The excess rainfall intensity is allowed to vary in time, however.
2. A watershed can have many UHs, each corresponding to a given rainfall duration. So we can have say 1 h, 2 h, and 4 h UHs, excited by 1 h 1 cm/h, 2 h 0.5 cm/h, and 4 h 0.25 cm/h excess rainfalls, respectively. Once UH of a given duration is known, UH of any other duration may be determined using well-known *S*-curve procedure.
3. All discharge hydrographs derived from the UH of duration *D* have equal time base regardless of the rainfall intensity.

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4. UH is a linear transformation model of input rainfall to output runoff hydrograph. Another words, if the rainfall intensity of duration *D* (in h) is *X* (in cm/h), the output hydrograph is *XD* times the *D* hour unit hydrograph. Linearity in itself brings about the third characteristics mentioned above.

Let's focus more on the linearity assumption of UH theory. Consider discharge (Q) to be a function (f) of rainfall intensity (i). Function f is linear with respect to input i if for a general input such as $c_1 i_1 + c_2 i_2 + \cdots + c_n i_n$, we can write:

$$Q = f(c_1 i_1 + c_2 i_2 + \cdots + c_n i_n) \\ = c_1 f(i_1) + c_2 f(i_2) + \cdots + c_n f(i_n) \quad (1)$$

where c's are constants. Eq. (1) is often referred to as the principle of addivity or superposition. This principle follows that incremental hydrographs generated by consecutive intensities of a hyetograph can be added up when delayed by the proper time step. For a simple input ci, we have $Q = cf(i)$. This is often called the principle of proportionality in UH theory.

An important distinction in definition of nonlinearity must be made. Sivapalan et al. (2002) differentiated between linearity of discharge with respect to input rainfall (i) and that of a statistical property of discharge (Q) with respect to a geomorphological characteristic (such as area). Based on a simple linear model of flood response, they illustrated that the dynamical nonlinearity of the former case exist independently of the latter statistical nonlinearity. Our focus in this paper is on the dynamic nonlinearity.

Now, the question to ask is whether, contrary to UH assumption, watersheds exhibit nonlinearly in rainfall-runoff transformation. Many researchers have presented evidence on nonlinearity of (excess) rainfall-runoff response. Some have speculated that the nonlinearity in depth—discharge relationships is the source of nonlinearity in rainfall-runoff response. Others have used numerical simulations to study nonlinearity. Robinson et al. (1995), for example, showed that nonlinearity, be it dominated at small scale by hillslope response or at large scale by channel network hydrodynamics does not disappear at any scale. Yet others have attempted to interpret nonlinearity by studying watershed rainfall-runoff records. However, this latter interpretation runs into some difficulties. Estimation of excess rainfall from observed rainfall records is not straightforward and depends on how different rainfall losses are accounted for. Furthermore, extraction of direct runoff depends on the choice of base flow separation technique. As Nash and Sutcliffe (1970) stated, in real watersheds, proving linearity or nonlinearity is practically impossible since excess rainfall (as input) and direct runoff (as output) have been defined with ambiguity. As a result, it is often preferred to study nonlinearity in runoff response either based on the exact solution of flow hydraulics over simple plane geometries, or by simulation models, or by examining rainfall-runoff records in small impervious watersheds. Duband et al. (1993) also stressed the difficulties involved in delicate problem of baseflow separation that is common to many UH identification techniques.

Due to linearity assumption, UH application appears more appropriate to small watersheds since rainfall duration may be greater than equilibrium time. In theory, only peak discharge of an equilibrium hydrograph follows linear proportionality to excess rainfall intensity. As will be shown later, the rising limb of the discharge hydrograph does not follow linear dependence on rainfall intensity regardless of the watershed size. For much more dominant partial equilibrium hydrographs, it can be proved through application of kinematic wave theory on simple plane geometries that peak discharge is nonlinearly proportional to excess rainfall intensity. Moreover, it is known from theory and field observations that timing of the discharges arriving at the watershed outlet decrease in a nonlinear fashion with rainfall intensity. Saghafian et al. (2002), for example, showed that watershed travel time varies with rainfall intensity raised to a power equal to or smaller than 0.4. Although the chance of small watersheds reaching steady state over a wide range of rainfall intensity and duration is greater than larger watersheds, this ensures that only the peak discharges are a linear function of excess rainfall intensity in small watersheds. Another words, Eq. (1) is only valid for peak discharge at steady state. However, the timing and the discharge ordinates on the rising limb of the hydrograph do not vary linearly with the input rainfall intensity. Put differently, linear mapping (transformation) of hydrographs is not generally warranted. We will discuss this issue in more detail in the following sections.

Now, can one account for nonlinearity by generalizing the UH transformation? This paper attempts to find an analytic nonlinear way of transforming a given UH (or even a nonUH) hydrograph to other general hydrographs. Once a nonlinear approach to transform UH is introduced, the assumption of constant time base of the rising limb of the transformed hydrograph will also be relaxed. We will use the kinematic wave (KW) theory, recognized as a nonlinear model, as the basis for development of nonlinear UH transformation and to validate the proposed technique on simple watershed geometries. At first, the method is illustrated for a rectangular plane geometry and then evaluated for converging plane, diverging plane, and a small natural watershed. As a limitation of this study, only rising limb of the hydrograph is discussed.

Methodology

Consider a rectangular plane of length L, slope S, and roughness n subject to rainfall intensity i (Fig. 1). The relationship between flow depth h and unit discharge q for a general

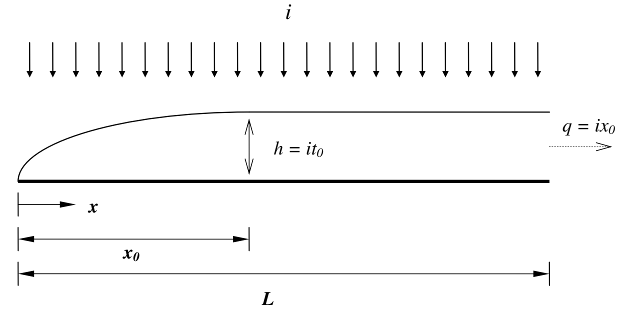

Figure 1 Water surface profile over a rectangular plane under rainfall intensity i at $t = t_0$.