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Stochastic modeling of transient stream–aquifer interaction with the nonlinear Boussinesq equation

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Summary In this article the effect of highly fluctuating stream stage on the adjacent alluvial valley aquifer is studied with a new analytical solution to the nonlinear transient groundwater flow equation subject to stochastic conductivity and time varying boundary conditions. A random conductivity field with known correlation structure represents uncertain heterogeneity. The resulting nonlinear stochastic Boussinesq equation is solved with the decomposition method. New expressions for the mean of the hydraulic head and its variance distribution are given. The procedure allows for the calculation of the mean head and error bounds in real situations when a limited sample allows the estimation of the conductivity mean and correlation structure only. Under these circumstances, the usual assumptions of a specific conductivity probability distribution, logarithmic transformation, small perturbation, discretization, or Monte Carlo simulations are not possible. The solution is verified via an application to the Scioto River aquifer in Ohio, which suffers from periodic large fluctuations in river stage from seasonal flooding. Predicted head statistics are compared with observed heads at different monitoring wells across the aquifer. Results show that the observed transient water table elevation in the observation well lies in the predicted mean plus or minus one standard deviation bounds. The magnitude of uncertainty in predicted head depends on the statistical properties of the conductivity field, as described by its coefficient of variability and its correlation length scale.

Introduction

The study of stream–aquifer hydraulics is of great interest as several flow and contaminant problems can be modeled, understood and quantified. The quantification of the hydraulics of the stream–aquifer in an alluvial valley require a good knowledge of the controlling input hydrogeo-

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logical parameters, such as hydraulic conductivity, specific yield, recharge, as well as boundary effects that are associated with the stream. Small changes in the stream elevation can cause a large variation in the groundwater elevation in the aquifer. The spread of contaminants in stream–aquifer systems from the river to the aquifer or from the aquifer to the river is also of concern. The hydraulics of such systems has been studied in the literature from both a deterministic point of view as well a stochastic point of view.

Stream–aquifer systems can be quantitatively studied using Laplace's equation subject to nonlinear free surface boundary conditions and time dependent river boundary conditions. Strack (1989) has shown that when Dupuit assumptions of negligible vertical flow, as compared to horizontal, are valid the nonlinear Boussinesq equation is a viable alternative to Laplace's equation. For the Boussinesq equation the vertical coordinate does not exist and the free surface boundary condition is not needed, and hence time dependent boundary conditions can be easily incorporated into the analysis. The hydraulics of stream–aquifer systems have been studied by several researchers when the input controlling parameters are known with certainty using both analytical and numerical methods (Polubarinova-Kochina, 1962; Kirkham, 1966). The solution to Laplace's equation with free surface boundary conditions was compared with the solution of the linearized Boussinesq equation for the case of sudden drawdown in the river levels and for uniform and nonuniform rainfall (Van De Giesen et al., 1994). Tabidian et al., 1992 studied groundwater level fluctuations for changes in the stage levels in the associated streams. Hussein and Schwartz (2003) have studied the coupled groundwater–surface water problems and quantified the expected transport in both the aquifer and the stream. The coupled canal flow–groundwater flow equation has been solved analytically and the results are compared with the numerical model, MODFLOW, for small changes in the water level disturbances (Lal, 2000). Moench and Barlow (2000) have solved analytically the one-dimensional flow equation in confined and leaky aquifers and the two-dimensional flow equation in a plane perpendicular to the stream in phreatic aquifers where the stream is assumed to penetrate the full thickness of the aquifer.

Most stream–aquifer systems can be viewed as heterogeneous geological media with the governing flow equations being described as stochastic equations. When the aquifer parameters and the boundary conditions are not known with certainty, the preferred way to model the system has been stochastically. Using the stochastic approach, stream–aquifer models have been solved both analytically as well as numerically. Weissmann and Fogg (1999) and Weissmann et al. (1999) have studied the alluvial fan system from both deterministic as well as stochastic approaches, by modeling the large scale features deterministically and the intermediate heterogeneity using transition probability geostatistics. Hantush and MariZo (1997, 2002) have also studied the flow equations from a stochastic point of view.

The stream–aquifer interaction problem with time varying boundaries has been solved by Workman et al., 1997) using the deterministic linearized Boussinesq equation. In a later study, Serrano and Workman (1998) solved the same problem using the nonlinear Boussinesq equation and Adomian's method of decomposition. Decomposition is now being used to solve deterministic, stochastic, linear or nonlinear equations in various branches of science and engineering (Adomian, 1991, 1994; Srivastava and Singh, 1999; Biazar et al., 2003; Wazwaz, 2000; Wazwaz and Gorguis, 2004; Srivastava, 2005). In groundwater flow problems, this method has been extensively used (Serrano, 1992, 1995a,b, 2003; Serrano and Unny, 1987; Serrano and Adomian, 1996; Adomian and Serrano, 1998), to obtain analytical, and sometimes closed-form, solutions to linear, nonlinear and stochastic problems. The method has been shown to be systematic, robust, and sometimes capable of handling large variances in the controlling hydrogeological parameters.

In this paper the work of Serrano and Workman (1998) has been extended to incorporate heterogeneity in the hydraulic conductivity represented stochastically. The nonlinear transient groundwater flow equation subject to stochastic conductivity and time varying boundary conditions is solved using decomposition. A random conductivity field with known correlation structure represents uncertain heterogeneity. Since decomposition does not require the assumptions of normality or smallness, we consider the practical situation when only the mean and correlation of the hydraulic conductivity are given based on a limited set of field samples. This is a common scenario in hydrologic practice. From the practical point of view, a modeling procedure that limits its application to Gaussian or any other conductivity fields is restrictive, since there is usually not enough information to ascertain the underlying probability distribution. A more common scenario in applications is one when the modeler possesses a limited sample from which only the first two moments can be estimated. Thus, a modeling procedure that allows the inclusion of these two measures, regardless of the underlying density function, appears practical. In this paper the solution and its second-order statistics are verified via an application to the Scioto River aquifer in Ohio. Predicted head statistics are compared with observed heads at different monitoring wells across the aquifer.

Analytical solution to the stochastic transient groundwater flow equation in unconfined aquifers

The groundwater flow-equation in a horizontal unconfined aquifer of length l_x with Dupuit assumptions given by Bear (1979) as

$$\frac{\partial h}{\partial t} - \frac{1}{S}\frac{\partial}{\partial x}\left(Kh\frac{\partial h}{\partial x}\right) = \frac{R}{S} \quad 0 \leqslant x \leqslant l_x \text{ and } 0 < t \tag{1}$$

where $h(x, t)$ is the hydraulic head (m), K is the hydraulic conductivity (m/day), R is the recharge (m/day), l_x is the length of the aquifer (m), x is the spatial coordinate (m), and t is the time coordinate (day).

The boundary conditions imposed on (1) are

$$\begin{aligned} h(0,t) &= H_1(t) \\ h(l_x,t) &= H_2(t) \\ h(x,0) &= H_0(x) \end{aligned} \tag{2}$$

where $H_1(t)$ and $H_2(t)$ are time fluctuating heads at the left and the right boundaries (m). $H_0(x)$ is the initial head across the aquifer (m).