Available online at www.sciencedirect.com

ScienceDirect

Expo. Math. 34 (2016) 62–81

EXPOSITIONES MATHEMATICAE

www.elsevier.com/locate/exmath

Destabilization

S. Kaliszewski, Tron Omland, John Quigg*

School of Mathematical and Statistical Sciences, Arizona State University, Tempe, AZ 85287, United States

Received 3 March 2015; received in revised form 18 August 2015

Abstract

This partly expository paper first supplies the details of a method of factoring a stable C^*-algebra A as $B \otimes \mathcal{K}$ in a canonical way. Then it is shown that this method can be put into a categorical framework, much like the crossed-product dualities, and that stabilization gives rise to an equivalence between the nondegenerate category of C^*-algebras and a category of "$\mathcal{K}$-algebras". We consider this equivalence as "inverting" the stabilization process, that is, a "destabilization".

Furthermore, the method of factoring stable C^*-algebras generalizes to Hilbert bimodules, and an analogous category equivalence between the associated enchilada categories is produced, giving a destabilization for C^*-correspondences.

Finally, we make a connection with (double) crossed-product duality.
© 2015 Elsevier GmbH. All rights reserved.

MSC 2010: primary 46L05; secondary 46L06; 46L08; 46M15

Keywords: Stabilization; Compact operators; Category equivalence; C^*-correspondence

1. Introduction

We start with some well-known facts that nowadays can be found in any textbook on C^*-algebras. The *stabilization* of a C^*-algebra A is $A \otimes \mathcal{K}$, where $\mathcal{K}$ denotes the compact

* Corresponding author.

E-mail addresses: kaliszewski@asu.edu (S. Kaliszewski), omland@asu.edu (T. Omland), quigg@asu.edu (J. Quigg).

http://dx.doi.org/10.1016/j.exmath.2015.10.003
0723-0869/© 2015 Elsevier GmbH. All rights reserved.

operators on a separable infinite-dimensional Hilbert space. Since $\mathcal{K} \otimes \mathcal{K} \simeq \mathcal{K}$, stabilizing twice does not (up to isomorphism) produce anything new, as the name suggests. In other words, stabilizations of C^*-algebras, are precisely those C^*-algebras A such that $A \simeq A \otimes \mathcal{K}$, and any A satisfying this property is simply called *stable*. An obvious question is: how can we characterize stable C^*-algebras? While this problem may have several approaches, our goal is to answer the following: given a C^*-algebra A, how can we decide whether there exists *some* C^*-algebra B such that $A \simeq B \otimes \mathcal{K}$, and then how can we produce such a B in a canonical way? A trivial answer comes straight from the definition, namely there exists such a B if and only if $A \simeq A \otimes \mathcal{K}$, in which case we can take $B = A$. This is unsatisfying on two levels. First of all, the property $A \simeq A \otimes \mathcal{K}$ is not very convenient to check, and secondly, the choice $B = A$ does not allow for a possibly unstable algebra B. Another, possibly more useful, characterization of stable C^*-algebras that seems to us to be folklore is that A is stable if and only if there is a nondegenerate copy of $\mathcal{K}$ in $M(A)$. Then one can use a choice of matrix units in $\mathcal{K}$ to decompose A as infinite matrices whose entries come from a C^*-algebra B, and then $A \simeq B \otimes \mathcal{K}$. The challenge is to produce a C^*-algebra that is isomorphic to this B without having to choose matrix units, i.e., canonically. One way becomes apparent by considering how to pick B out of $B \otimes \mathcal{K}$. We have an injection from B to $M(B \otimes \mathcal{K})$ given by $b \mapsto b \otimes 1_{\mathcal{K}}$, where $1_{\mathcal{K}}$ denotes the identity element of $M(\mathcal{K})$. The trick is to identify the image $B \otimes 1_{\mathcal{K}}$ inside $M(B \otimes \mathcal{K})$. Obviously $B \otimes 1_{\mathcal{K}}$ commutes with $1_B \otimes \mathcal{K}$, and also multiplies $1_B \otimes \mathcal{K}$ into $B \otimes \mathcal{K}$. This gives the characterization of interest to us, and a short, very rough, summary (using different arguments from those we present here) can be found in [4, Section 3]. We feel that it is useful to "officially" record the details for convenient reference, since it seems difficult to dig them out of the literature, and moreover we think it is appropriate to make our arguments as elementary as possible. We give the details in Proposition 3.4, after recalling some background material in Section 2.

In Theorem 4.4 we parlay the characterization of Proposition 3.4 into an equivalence between the categories of C^*-algebras and of "$\mathcal{K}$-algebras" (stable C^*-algebras equipped with a given embedding of $\mathcal{K}$ — see Section 3 for the definitions). Here the morphisms in both categories involve nondegenerate homomorphisms into multiplier algebras. We then discuss how this category equivalence fits into a general framework we described in [7, Section 4]: if we consider the stabilization process $B \mapsto A = B \otimes \mathcal{K}$, Theorem 4.4 tells us what extra data we need to recover B from A, i.e., to "invert the process". In [7] we study this in a technical manner, using basic category theory, and we introduce a concept we call "good inversion", where the inverse image of an output of the process is classified up to isomorphism by the automorphisms of the output. We observe in Section 4 that Theorem 4.4 is an example of a good inversion.

In Section 5 we extend Proposition 3.4 to Hilbert bimodules, and in Section 6 we apply this to make stabilization into an equivalence between enchilada[1] categories, where the morphisms come from C^*-correspondences.

Recall that two C^*-algebras A and B are *stably isomorphic* if $A \otimes \mathcal{K} \simeq B \otimes \mathcal{K}$, and this condition is stronger than Morita equivalence, that is, *every* C^*-algebra A is stable up to

[1] The term "enchilada" is informal, and originated when the authors of the AMS Memoir [3] published a smaller paper [2] as an introduction to the techniques, and referred to the smaller paper as the "little taco" and the memoir as the "big enchilada".

Download English Version:

https://daneshyari.com/en/article/4582354

Download Persian Version:

https://daneshyari.com/article/4582354

Daneshyari.com