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Expo. Math. 25 (2007) 309 – 323
EXPOSITIONES MATHEMATICAE
www.elsevier.de/exmath

On composition operators acting between Hardy and weighted Bergman spaces

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Received 10 November 2006

Abstract

We present a unified approach to some known and some new criteria for the boundedness and compactness of composition operators mapping a weighted Bergman space A^p_α into another weighted Bergman space A^q_β, where $q \geqslant p$ and $\alpha, \beta > -1$, also obtaining some asymptotic formulas for the essential norm. The results are also valid in the limit cases when at least one of the spaces is a Hardy space (i.e., when α or $\beta = -1$) and complement the existing results by various authors.
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MSC 2000: primary 47B33secondary 30H05

Keywords: Hardy spaces; Bergman spaces; Composition operator; Compactness; Essential norm; Nevanlinna counting function

1. Introduction and main results

Let $\mathbb{D}$ denote the open unit disc in the complex plane and $\mathbb{T}$ its boundary, the unit circle. We will use the notation $H(\mathbb{D})$ for the algebra of all analytic functions on $\mathbb{D}$.

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doi:10.1016/j.exmath.2007.03.001

An *analytic self-map* φ of $\mathbb{D}$ is a function in $H(\mathbb{D})$ such that $\varphi(\mathbb{D}) \subset \mathbb{D}$. Every such map induces the *composition operator* C_φ acting on $H(\mathbb{D})$, defined by $C_\varphi(f) = f \circ \varphi$. It is well known that every composition operator is a bounded linear operator on any of the standard Hardy and Bergman spaces of $\mathbb{D}$. This had essentially appeared already in Littlewood's paper [14]; see also [9, Chapter 1]. The first papers in the modern spirit of operators acting on function spaces were [16,20]. The monographs [6,22] present further developments and give an excellent overview of the subject up to the early or mid 1990's.

A composition operator acting on other spaces of analytic functions, or between two different spaces, need not be bounded. Thus, the first and most natural question that arises is that of characterizing all possible bounded operators in terms of their symbols. Such conditions can be of either geometric or analytic nature (see [6,19,24]). One can find many instances of this research in the literature, not only for the composition operators but also for the closely related multiplication operators and for the weighted composition operators that generalize both the composition and the multiplication operators (see, for example, [4,5] or [7,8]).

The purpose of this paper is to present different characterizations of both the bounded and the compact composition operators acting between two function spaces, each of which can be a Hardy space or a weighted Bergman space, all possible combinations being admitted. This complements part of the recent results on the more general weighted composition operators between two weighted Bergman spaces [7,8]. Our approach is somewhat different from the one taken by Čučković and Zhao. We hardly use any operator theory and, in addition to the generalized Carleson measures and Berezin transform, we also emphasize the role of the generalized Nevanlinna counting function in the spirit of Smith's work [24–26]. Also, our method allows us to cover the operators from Hardy into Bergman spaces, thus extending their results to this case as well. We now review the motivation and describe our main results.

Recall that a linear operator is said to be *compact* if it takes bounded sets to relatively compact sets. In the case of composition operators acting between two Banach spaces of analytic functions, this has several equivalent formulations. The study of compactness of composition operators began on the Hardy space H^2 in the pioneering work by Shapiro and Taylor [23], achieving its high point in a paper by Shapiro [21] where, among other results, a formula for the essential norm of C_φ acting on H^2 was found. An important ingredient, both there and in our study, is the use of $N_{\varphi,\gamma}$, the generalized Nevanlinna counting function associated with φ and defined as follows:

$$N_{\varphi,\gamma}(w) = \sum_{z \in \varphi^{-1}\{w\}} \left(\log \frac{1}{|z|}\right)^\gamma, \quad w \in \mathbb{D} \backslash \{\varphi(0)\}, \quad \gamma > 0,$$

where $z \in \varphi^{-1}\{w\}$ is repeated according to the multiplicity of the zero of $\varphi - w$ at z. (In [21], the case $\gamma = 1$ was considered.)

MacCluer and Shapiro [15] showed that C_φ is compact on the Bergman space A^p if and only if φ has no finite angular derivative at any point on the unit circle (see also [6]). Later on, bounded and compact composition operators from the weighted Bergman space A^p_α into A^q_β, $p \leqslant q$, were characterized in terms of the generalized Nevanlinna counting function by Riedl (the case $\alpha = \beta = -1$, understanding in this limit case that $A^p_{-1} = H^p$, the Hardy

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