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Resolvable generalized difference matrices: Existence and applications ☆

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Difference matrices, elsewhere also called difference schemes, form a useful tool in the construction of various interesting combinatorial objects such as orthogonal arrays. In this paper, we introduce the concept of a resolvable generalized difference matrix (RGDM) of strength t. The task of the paper is to study the existence and applications of RGDMs. As a result, many new classes of RGDMs are presented. In addition, some approaches of constructing 2-compatible CDPs by using RGDMs of strength three are established. With those constructions, we are able to make a big improvement on the known existence of orbit-disjoint CDPs.

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1. Introduction

Let G be an abelian group of order g whose operation is written as addition. For an arbitrary positive integer t, let G^t be the t-fold direct product of G with itself. By $diag(G^t)$ or G_0^t we will mean the diagonal subgroup of G^t, that is the group of G^t of the t-tuples with all coordinates equal. The cosets of this subgroup will be denoted by G_i^t, $i = 0, 1, \cdots, g^{t-1} - 1$. In most occasions of this paper, G is taken as the additive group of $\mathbb{Z}_g$. Throughout, we will use $\mathbb{Z}_g = \{0, 1, \cdots, g-1\}$ to denote the residual-class ring of integers modulo g.

Definition 1.1. For the given positive integers t, w, g and λ with $w \geq 3$, a $w \times \lambda g^{t-1}$ matrix M with entries from G is termed a *difference matrix* (DM) of strength t, denoted as a $(g, w, \lambda; t)$-DM over G, if the columns of any $t \times \lambda g^{t-1}$ submatrix of M are evenly distributed over the cosets of $diag(G^t)$ in G^t.

Clearly, a $(g, w, \lambda; t)$-DM over G is also a $(g, w, \lambda g; t-1)$-DM over G from Definition 1.1. Here we present a $(5, 4, 1; 3)$-DM over $\mathbb{Z}_5$ as an example below.

Example 1.2. Let $t = 3$, $k = 4$, $\lambda = 1$ and $g = 5$, then a $(5, 4, 1; 3)$-DM over $\mathbb{Z}_5$ is depicted as follows:

$$\begin{pmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&2&3&4&0&1&2&3&4&0&1&2&3&4&0&1&2&3&4&0&1&2&3&4 \\ 0&2&4&1&3&1&3&0&2&4&2&4&1&3&5&3&0&2&4&1&4&1&3&0&2 \\ 0&3&1&4&2&1&4&2&0&3&2&0&3&1&4&3&1&4&2&0&4&2&0&3&1 \end{pmatrix} \qquad \square$$

Note that a difference matrix of strength 2 in Definition 1.1 is just the classic notion of a difference matrix which is a matrix with entries from an additive group G satisfying the property that all elements of G appear in the difference of any two distinct rows the same number of times. For more information on the classic difference matrix, the reader may refer to [12,16] and references therein. In addition, the definition of a difference matrix can be easily extended to non-abelian groups. Also note that a difference matrix of strength $t = 1$ is trivial and of uninteresting. In what follows, the parameter t is always assumed to be not less than two, unless otherwise stated.

Difference matrices, elsewhere also called difference schemes, were introduced in a seminal paper by Bose and Bush [5] in the case strength $t = 2$. They introduced and studied these matrices to facilitate the construction of orthogonal arrays. Over the past decades, difference matrices have been the subject of much study. They have played a prominent role not only in design theory but also in statistics. Difference matrices have also been used for the construction of cyclic designs related to optical orthogonal codes. There exists a vast literature on the construction and application of difference matrices (see, for example, [8,11–13,19,21,31] and the references therein). A large number of known DMs of strength $t = 2$ are well documented in [12]. The generalization of a difference