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Repeated-root constacyclic codes of length $3lp^s$ and their dual codes ☆

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A B S T R A C T

Let $p \neq 3$ be any prime and $l \neq 3$ be any odd prime with $\gcd(p, l) = 1$. The multiplicative group $F_q^* = \langle \xi \rangle$ can be decomposed into mutually disjoint union of $\gcd(q-1, 3lp^s)$ cosets over the subgroup $\langle \xi^{3lp^s} \rangle$, where ξ is a primitive $(q-1)$th root of unity. We classify all repeated-root constacyclic codes of length $3lp^s$ over the finite field F_q into some equivalence classes by this decomposition, where $q = p^m$, s and m are positive integers. According to these equivalence classes, we explicitly determine the generator polynomials of all repeated-root constacyclic codes of length $3lp^s$ over F_q and their dual codes. Self-dual cyclic codes of length $3lp^s$ over F_q exist only when $p = 2$. We give all self-dual cyclic codes of length $3 \cdot 2^s l$ over F_{2^m} and their enumeration. We also determine the minimum Hamming distance of these codes when $\gcd(3, q-1) = 1$ and $l = 1$.

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1. Introduction

Constacyclic codes over finite fields play a very important role in the theory of error-correcting codes. More importantly, constacyclic codes have practical applications. As these codes have rich algebraic structures, they can be efficiently encoded and decoded using shift registers. They also have very good error-correcting properties. All of those explain their preferred role in engineering.

Repeated-root cyclic codes were first investigated in the 1990s by Castagnoli in [1] and Van Lint in [2], where it was proved that repeated-root cyclic codes have a concatenated construction, and are asymptotically bad. However, it is well known that there still exist a few optimal such codes (see [15–17]), which encourages many scholars to study the class of codes. For example, Dinh determined the generator polynomials of all constacyclic codes and their dual codes over F_q of length $2p^s$, $3p^s$ and $6p^s$ in [3–5]. Since then, these results have been extended to more general code lengths. In 2012, Bakshi and Raka gave the generator polynomials of all constacyclic codes of length 2^tp^s over F_q in [6], where q is a power of an odd prime p. In the same year, Chen et al. studied repeated root constacyclic codes of length l^tp^s over F_q in [9]. In 2014, Chen et al. studied all constacyclic codes of length lp^s over F_q in [7], where l is a prime different from p. In [7], all constacyclic codes of length lp^s over F_q and their dual codes were obtained, and all self-dual and all linear complementary dual constacyclic codes were given. In 2015, Raka studied repeated root constacyclic codes of length l^tp^s over F_q in [10]. Recently, in [8], Sharma explicitly determined the generator polynomials of all repeated-root constacyclic codes of length l^tp^s over F_{p^m} and their dual codes. Further, they listed all self-dual cyclic and negacyclic codes and also determined all self-orthogonal cyclic and negacyclic codes of length l^tp^s over F_{p^m}. What's more, Sharma and Rani gave a method to compute repeated root constacyclic codes of length np^s over F_q, for any positive integer n coprime to p, however the generator polynomials are not determined explicitly, in [11]. Chen et al. studied all constacyclic codes of length $2l^mp^s$ over F_q of characteristic p in [12], and they gave the characterization and enumeration of all linear complementary dual and self-dual constacyclic codes of length $2l^mp^s$ over F_q. In the conclusion of their paper, they proposed to study all constacyclic codes of length kl^mp^s over F_q, where p is the characteristic of F_q, l is an odd prime different from p, and k is a prime different from l and p. However, this is not an easy work.

In this paper, we study all constacyclic codes of length $3lp^s$ over F_q, where $p \neq 3$ is any prime and $l \neq 3$ is any odd prime with $\gcd(p,l) = 1$. The article is organized as follows. In Section 3, we decompose the multiplicative cyclic group $F_q^* = \langle \xi \rangle$ into mutually disjoint union of cosets of $\langle \xi^{3lp^s} \rangle$ and show that λ-constacyclic code is equivalent to μ-constacyclic code if λ and μ lie in the same coset. Based on this decomposition, Section 4 explicitly determines the generator polynomials of all λ-constacyclic codes of length $3lp^s$ over F_q and their dual codes, where λ is any none-zero element of F_q and $q = p^m$ is a power of prime. As an application, we give all self-dual cyclic codes of length $3 \cdot 2^sl$ over F_{2^m} and their enumeration in Section 5. In Section 6, we also