

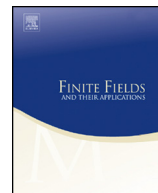


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On maximal curves that are not quotients of the Hermitian curve



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ABSTRACT

For each prime power ℓ the plane curve \mathcal{X}_ℓ with equation $Y^{\ell^2-\ell+1} = X^{\ell^2} - X$ is maximal over \mathbb{F}_{ℓ^6} . Garcia and Stichtenoth in 2006 proved that \mathcal{X}_3 is not Galois covered by the Hermitian curve and raised the same question for \mathcal{X}_ℓ with $\ell > 3$; in this paper we show that \mathcal{X}_ℓ is not Galois covered by the Hermitian curve for any $\ell > 3$. Analogously, Duursma and Mak proved that the generalized GK curve \mathcal{C}_{ℓ^n} over $\mathbb{F}_{\ell^{2n}}$ is not a quotient of the Hermitian curve for $\ell > 2$ and $n \geq 5$, leaving the case $\ell = 2$ open; here we show that \mathcal{C}_{2^n} is not Galois covered by the Hermitian curve over $\mathbb{F}_{2^{2n}}$ for $n \geq 5$.

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1. Introduction

Let \mathbb{F}_{q^2} be the finite field with q^2 elements, where q is a power of a prime p , and let \mathcal{X} be an \mathbb{F}_{q^2} -rational curve, that is a projective, absolutely irreducible, non-singular algebraic curve defined over \mathbb{F}_{q^2} . \mathcal{X} is called \mathbb{F}_{q^2} -maximal if the number $\mathcal{X}(\mathbb{F}_{q^2})$ of its \mathbb{F}_{q^2} -rational points attains the Hasse–Weil upper bound

$$q^2 + 1 + 2gq,$$

where g is the genus of \mathcal{X} . Maximal curves have interesting properties and have also been investigated for their applications in Coding Theory. Surveys on maximal curves are found in [9–11,13,32,33] and [23, Chapt. 10].

The most important example of an \mathbb{F}_{q^2} -maximal curve is the Hermitian curve \mathcal{H}_q , defined as any \mathbb{F}_{q^2} -rational curve projectively equivalent to the plane curve with Fermat equation

$$X^{q+1} + Y^{q+1} + T^{q+1} = 0.$$

The norm-trace equation

$$Y^{q+1} = X^q T + X T^q$$

gives another model of \mathcal{H}_q , \mathbb{F}_{q^2} -equivalent to the Fermat model, see [15, Eq. (2.15)]. For fixed q , \mathcal{H}_q has the largest possible genus $g(\mathcal{H}_q) = q(q - 1)/2$ that an \mathbb{F}_{q^2} -maximal curve can have. The automorphism group $\text{Aut}(\mathcal{H}_q)$ is isomorphic to $\text{PGU}(3, q)$, the group of projectivities of $\text{PG}(2, q^2)$ commuting with the unitary polarity associated with \mathcal{H}_q .

By a result commonly attributed to Serre, see [26, Prop. 6], any \mathbb{F}_{q^2} -rational curve which is \mathbb{F}_{q^2} -covered by an \mathbb{F}_{q^2} -maximal curve is also \mathbb{F}_{q^2} -maximal. In particular, \mathbb{F}_{q^2} -maximal curves are given by the Galois \mathbb{F}_{q^2} -subcovers of an \mathbb{F}_{q^2} -maximal curve \mathcal{X} , that is by the quotient curves \mathcal{X}/G over a finite \mathbb{F}_{q^2} -automorphism group $G \leq \text{Aut}(\mathcal{X})$.

Most of the known maximal curves are Galois subcovers of the Hermitian curve, many of which were studied in [4,5,15]. Garcia and Stichtenoth [14] discovered the first example of maximal curve not Galois covered by the Hermitian curve, namely the curve $Y^7 = X^9 - X$ maximal over \mathbb{F}_{36} . It is a special case of the curve \mathcal{X}_ℓ with equation

$$Y^{\ell^2 - \ell + 1} = X^{\ell^2} - X, \tag{1}$$

which is \mathbb{F}_{ℓ^6} -maximal for any $\ell \geq 2$; see [1]. In [17], Giulietti and Korchmáros showed that the Galois covering of \mathcal{X}_ℓ given by

$$\begin{cases} Z^{\ell^2 - \ell + 1} = Y^{\ell^2} - Y \\ Y^{\ell + 1} = X^\ell + X \end{cases}$$

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