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Repeated-root constacyclic codes of length kl^ap^b over a finite field

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ABSTRACT

In this paper, we give an sufficient and necessary condition that a polynomial is irreducible over $\mathbb{F}_q$. For different odd primes k, l and p, we obtain generator polynomials of constacyclic codes of length kl^ap^b over finite field $\mathbb{F}_q$, where $\mathrm{char}\mathbb{F}_q = p$ and $(\mathrm{ord}_k q, l) = 1$. And we answer the question in [3] under the condition.

1. Introduction

Let $\mathbb{F}_q$ be the finite field of size q, where q is a power of p. For $\lambda \in \mathbb{F}_q^*$, a λ-constacyclic code of length n over $\mathbb{F}_q$ can be viewed as an ideal $\langle g(x) \rangle$ in the ring $\frac{\mathbb{F}_q[x]}{x^n-\lambda}$, where the generator polynomial $g(x)$ is the unique monic polynomial of minimum degree in the code and $g(x)|(x^n - \lambda)$. The ring $\frac{\mathbb{F}_q[x]}{x^n-\lambda}$ is a principal ideal ring, i.e., every ideal can be

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generated by a monic divisor of $x^n - \lambda$. So the irreducible factorization of $x^n - \lambda$ in $\mathbb{F}_q[x]$ determines all λ-constacyclic codes of length n. When $p \nmid n$, a λ-constacyclic code is called a (simple-root) constacyclic code. When $p|n$, a λ-constacyclic code is called a repeated-root constacyclic code.

Recently, Dinh determined generator polynomials of all constacyclic codes over $\mathbb{F}_q$, of lengths $2p^s$, $3p^s$, lp^s and $2l^m p^n$ [5–8]. Generator polynomials of all repeated-root constacyclic codes of length $2^t p^s$ over $\mathbb{F}_q$ were given in [1]. In [2] and [4], authors studied all repeated-root constacyclic codes of length lp^s over $\mathbb{F}_q$.

In this paper, we study all repeated-root constacyclic codes of length $kl^a p^b$ over $\mathbb{F}_q$, where k, l, p are different odd primes, $(\mathrm{ord}_k q, l) = 1$ and p is the character of $\mathbb{F}_q$. We give generator polynomials of all repeated-root constacyclic codes. In section 2, we give some results about n-equivalent relation introduced by B. Chen et al. in [2]. And we also give some sufficient and necessary conditions that a polynomial is irreducible over $\mathbb{F}_q$. In section 3, we give our main results, the factorization of $x^{kl^a p^b} - \lambda$ in $\mathbb{F}_q[x]$ and generator polynomials of all repeated-root constacyclic codes of length $kl^a p^b$ over $\mathbb{F}_q$. At last, we give an example of repeated-root constacyclic codes of length $5 \times 3^2 \times 7$ over $\mathbb{F}_7$.

2. Preliminaries

For classifying constacyclic codes of length n over $\mathbb{F}_q$, B. Chen et al. introduced the equivalence relation $\sim_n$ called n-equivalence in [2]. These constacyclic codes belonging to the same equivalence class have the same distance structure and the same algebraic structure.

Definition 1. [2] Let n be a positive integer. For any elements $\lambda, \mu \in \mathbb{F}_q^*$, if the polynomial $\lambda x^n - \mu$ has a root in $\mathbb{F}_q$, λ and μ are called *n-equivalent* in $\mathbb{F}_q^*$, denoted by $\lambda \sim_n \mu$.

Proposition 1. *[3] For any $\lambda, \mu \in \mathbb{F}_q^*$, the following four statements are equivalent:*

(1) $\lambda^{-1}\mu \in \langle \beta^n \rangle$.

(2) $(\lambda^{-1}\mu)^d = 1$, where $d = \frac{q-1}{(n,q-1)}$.

(3) λ and μ are n-equivalent in $\mathbb{F}_q^*$, namely there exists an element $a \in \mathbb{F}_q^*$ such that $a^n \lambda = \mu$.

(4) There exists an $a \in \mathbb{F}_q^*$ such that

$$\varphi_a : \mathbb{F}_q[x]/\langle x^n - \mu \rangle \to \mathbb{F}_q[x]/\langle x^n - \lambda \rangle$$
$$f(x) \mapsto f(ax)$$

is an $\mathbb{F}_q$-algebra isomorphism.

Next we give some results on irreducible polynomials.