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Repeated-root constacyclic codes of length $4\ell^m p^n$

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ABSTRACT

Constacyclic codes form a well-known class of linear codes, and are generalizations of cyclic and negacyclic codes. In this paper, we determine generator polynomials of all constacyclic codes of length $4\ell^m p^n$ over the finite field $\mathbb{F}_q$ with q elements, where p, ℓ are distinct odd primes, q is a power of p and m, n are positive integers. We also determine their dual codes, and list all self-dual constacyclic codes of length $4\ell^m p^n$ over $\mathbb{F}_q$.

1. Introduction

Constacyclic codes form an algebraically-rich family of error-correcting codes, and have good error-correcting properties. These codes can be effectively encoded and decoded using linear shift registers, which justify their preferred role from engineering perspective.

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In [4], constacyclic codes are first introduced as generalizations of cyclic and negacyclic codes. Since then, the problem of determination of the algebraic structure of constacyclic codes is of great interest. However, it is known only for some special classes of constacyclic codes. Below we provide a brief survey of some of the recent results in this direction.

In a series of papers, Dinh [11–15] determined all repeated-root constacyclic codes of length ap^s over $\mathbb{F}_{p^r}$ (p is a prime) in terms of their generator polynomials, where $a \in \{1,2,3,4,6\}$, p is a prime with $\gcd(a,p)=1$ and s,r are positive integers. Later, Bakshi and Raka [2] determined all Λ-constacyclic codes of length 2^tp^s ($t \geq 1, s \geq 0$ are integers) over $\mathbb{F}_{p^r}$, where p is an odd prime and Λ is a non-zero element in $\mathbb{F}_{p^r}$ whose multiplicative order is 2^a with $a \geq 0$ an integer. Chen et al. [9] defined an equivalence relation, called isometry, to characterize generator polynomials of all constacyclic codes of length ℓ^tp^s over $\mathbb{F}_{p^r}$, where ℓ, p are distinct primes and s,t,r are positive integers. In another work, Chen et al. [7] characterized all constacyclic codes of length ℓp^s over $\mathbb{F}_{p^r}$ and their dual codes, where ℓ, p are distinct primes and s,r are positive integers. They also determined all self-dual and complimentary-dual constacyclic codes of length ℓp^s over $\mathbb{F}_{p^r}$. Sharma [19] determined all constacyclic codes of length ℓ^tp^s over $\mathbb{F}_{p^r}$ and their dual codes, where ℓ, p are distinct primes, ℓ is odd and s,t,r are positive integers. She also determined all self-dual and self-orthogonal constacyclic codes of length ℓ^tp^s over $\mathbb{F}_{p^r}$. Recently, Chen et al. [8] determined the algebraic structure of all constacyclic codes of length $2\ell^tp^s$ over $\mathbb{F}_{p^r}$ and their dual codes in terms of their generator polynomials, where ℓ, p are distinct odd primes and s,t,r are positive integers. They also determined all complimentary-dual and self-dual constacyclic codes of length $2\ell^tp^s$ over $\mathbb{F}_{p^r}$. In another recent work, Batoul et al. [3] investigated the algebraic structure of constacyclic codes of length $a2^tp^s$ over $\mathbb{F}_{p^r}$ provided $p^r \equiv 1 \pmod{2^t}$, where p is an odd prime, a is an odd positive integer with $\gcd(a,p)=1$ and r,s,t are positive integers. They also provided certain sufficient conditions under which these codes are equivalent to cyclic codes of length $a2^tp^s$ over $\mathbb{F}_{p^r}$. In the same work, they derived a necessary and sufficient condition for the existence of a self-dual negacyclic code of length $a2^tp^s$ over $\mathbb{F}_{p^r}$. In another direction, Blackford [6] studied a class of simple-root constacyclic codes over finite fields that are isometric to their dual via a multiplier.

Throughout this paper, let p, ℓ be distinct odd primes, q be a power of the prime p, $\mathbb{F}_q$ be the finite field of order q and m, n be positive integers. In this paper, we determine the algebraic structure of all constacyclic codes of length $4\ell^mp^n$ over $\mathbb{F}_q$ and their dual codes in terms of their generator polynomials. Besides this, we determine all self-dual constacyclic codes of length $4\ell^mp^n$ over $\mathbb{F}_q$.

This paper is organized as follows: In Section 2, we state some preliminaries that are needed to derive our main results. In Section 3, we determine all repeated-root constacyclic codes of length $4\ell^mp^n$ over $\mathbb{F}_q$ and their dual codes by considering the following four cases separately: (i) $q \equiv 1 \pmod 4$ and $\gcd(\ell, q-1)=1$; (ii) $q \equiv 3 \pmod 4$ and $\gcd(\ell, q-1)=1$; (iii) $q \equiv 1 \pmod 4$ and $\gcd(\ell, q-1)=\ell$; and (iv) $q \equiv 3 \pmod 4$ and $\gcd(\ell, q-1)=\ell$ (Theorems 3.2–3.5). Finally, in Section 4, we determine all self-dual constacyclic codes of length $4\ell^mp^n$ over $\mathbb{F}_q$ (Theorem 4.1).