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Quasi-twisted codes with constacyclic constituent codes

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ABSTRACT

Quasi-twisted codes are generalizations of the familiar linear quasi-cyclic codes. In this paper, an algebraic method is used to investigate the relationship between quasi-twisted codes and constacyclic codes. Moreover, a generator polynomial of a quasi-twisted code with constacyclic constituent codes is given. Meanwhile, the conditions for a quasi-twisted code $\mathscr{C}$ of index ℓ and length ℓm to be equivalent to a constacyclic code of length ℓm are obtained. Finally, some examples are presented to illustrate the discussed results.

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1. Introduction

Quasi-twisted codes over finite fields form an important class of block codes that include constacyclic codes [3] and quasi-cyclic codes as special cases. Recently, quasi-twisted codes were studied in [1,5,8,9]. In [7], Ling and Solé viewed each quasi-cyclic code as a code over a polynomial ring, and extracted a description of each quasi-cyclic code as a direct sum of linear codes of shorter lengths over larger fields. These codes are called the constituent codes of the quasi-cyclic code in question. In [2], quasi-cyclic codes of length 5ℓ and index ℓ over $\mathbb{F}_q$ were obtained from a pair of codes over, respectively, $\mathbb{F}_q$ and $\mathbb{F}_{q^4}$, by a combinatorial construction called there the quintic construction. They are shown to be cyclic when the constituent codes are cyclic of odd length coprime to 5. In [6], Lim considered the same problem for quasi-cyclic codes of general index. In [4], Güneri and Özbudak also considered the same case. If the constituent codes of a quasi-cyclic code $\mathscr{C}$ of length $m\ell$ and index ℓ are cyclic, they proved that $\mathscr{C}$ can be viewed as a 2-D cyclic code of size $m \times \ell$ over $\mathbb{F}_q$. Moreover, in case that m and ℓ are also coprime to each other, $\mathscr{C}$ must be equivalent to a cyclic code.

In view of the analogy between cyclic and constacyclic codes on one hand and quasi-cyclic and quasi-twisted codes on the other hand, a natural question is to characterize quasi-twisted codes with constacyclic constituent codes. In this paper, we will apply an algebraic method to solve this problem and give the conditions that a quasi-twisted code is equivalent to a constacyclic code.

The material is organized as follows. The next section introduces a decomposition of constacyclic codes under the constashift. Section 3 considers the special class of quasi-twisted codes with constacyclic constituent codes. Section 4 considers the generator polynomial of the equivalent constacyclic code. Section 5 contains numerical examples. Section 6 concludes the paper.

2. The λ-circulant set decomposition of a λ-constacyclic code

Throughout this paper we require that $(m, q) = (\ell, q) = (m, \ell) = 1$ and $\lambda^{\ell+m-1} = 1$, where $\lambda \in \mathbb{F}_q^*$, $q = p^k$ for some positive integer k and p is a prime number. An $[n, k]_q$ code C is called λ-constacyclic if for each codeword $\mathbf{c} = (a_0, a_1, a_2, \ldots, a_{n-1}) \in C$, and $\lambda \in \mathbb{F}_q^*$, the vector $(\lambda a_{n-1}, a_0, a_1, \ldots, a_{n-2}) \in C$. In this section, we require that $(n, q) = 1$.

Definition 1. Let $\lambda \in \mathbb{F}_q^*$, and let C be a λ-constacyclic code of length n over $\mathbb{F}_q$, then a λ-*circulant matrix* A containing the codeword $(a_0, a_1, \ldots, a_{n-1})$ is defined as follows:

$$A = \begin{pmatrix} a_0 & a_1 & a_2 & \ldots & a_{n-1} \\ \lambda a_{n-1} & a_0 & a_1 & \ldots & a_{n-2} \\ \lambda a_{n-2} & \lambda a_{n-1} & a_0 & \ldots & a_{n-3} \\ \ldots & & \ldots & & \ldots \\ \lambda a_1 & \lambda a_2 & \lambda a_3 & \ldots & a_0 \end{pmatrix}.$$

Note that the row vectors of A are codewords of C.