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A critical review and some remarks about one- and two-weight irreducible cyclic codes

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ABSTRACT

A lot of work has already been done in one- and two-weight irreducible cyclic codes. In particular, in the remarkable work of Schmidt and White [8], all one- and two-weight irreducible cyclic codes were characterized, and it was conjectured that all of them are either subfield codes, or semiprimitive two-weight codes, or they belong to an exceptional set of eleven codes. In addition, the authors give, indirectly, a characterization of all semiprimitive two-weight irreducible cyclic codes over any finite field. The purpose of this work is to stress the importance of such characterization, and also, to present some remarks about one- and two-weight irreducible cyclic codes. This characterization and our remarks are important because through them we show that most of the recent results in one- and two-weight irreducible cyclic codes are, in one way or another, consequences of the work of Schmidt and White. For example, we will show that all new results regarding the two-weight irreducible cyclic codes, recently presented by C. Ding and J. Yang in [4], can be viewed as mere instances of such characterization. Moreover, through this characterization we generalize the results about projective two-weight irreducible cyclic codes that were presented by J. Wolfmann in [11]. Our results will also be useful to clarify that the family of two-weight irreducible cyclic codes presented by Rao and Pinnawala, in [7], is already included in the paper of Schmidt and White through an equivalence notion, and therefore,

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a major revision of the Schmidt and White conjecture is not needed.

1. Introduction

Cyclic codes are of great interest and importance because they can be efficiently implemented by using simple shift registers, and for this reason many practically important codes are cyclic. In addition, cyclic codes possess a rich algebraic structure that can be utilized in a variety of ways. A research topic of interest has been to use such rich algebraic structure in order to determine the weight distribution for either irreducible or reducible cyclic codes. With regard to the first ones, several authors have been working on this problem (see for example [5,8,11,3]). However, as was pointed out in [3], the problem of determining the weight distributions of irreducible cyclic codes is, in general, notoriously difficult. Fortunately, for the subfamily of one- and two-weight irreducible cyclic codes, a lot of work has already been done. In particular, in the remarkable work of Schmidt and White [8], all one- and two-weight irreducible cyclic codes were characterized, and it was conjectured that all of them are either subfield codes, or semiprimitive two-weight codes, or they belong to an exceptional set of eleven codes. In addition, the authors give, indirectly, a characterization of all semiprimitive two-weight irreducible cyclic codes over any finite field. The purpose of this work is to stress the importance of such characterization, and also, to present some remarks about one- and two-weight irreducible cyclic codes. This characterization and our remarks are important because through them we show that most of the recent results in one- and two-weight irreducible cyclic codes are, in one way or another, consequences of the work of Schmidt and White. For example, we will show that all new results regarding the two-weight irreducible cyclic codes, recently presented by C. Ding and J. Yang in [4], can be viewed as mere instances of such characterization. Moreover, through this characterization we generalize the results about projective two-weight irreducible cyclic codes that were presented by J. Wolfmann in [11]. Our results will also be useful to clarify that the family of two-weight irreducible cyclic codes presented by Rao and Pinnawala, in [7], is already included in the paper of Schmidt and White through an equivalence notion, and therefore, a major revision of the Schmidt and White conjecture is not needed.

With regard to the one-weight irreducible cyclic codes, we will show that the characterization recently presented in [4], by C. Ding and J. Yang is just an instance of the set of characterizations for the one-weight irreducible cyclic codes that was introduced by G. Vega in [9]. What is different here is that such set also shows that an irreducible cyclic code of dimension k over a finite field $\mathbb{F}_q$, with parity check polynomial $h(x)$, will be a one-weight cyclic code if and only if the least positive integer ρ for which x^ρ is congruent modulo $h(x)$ to some element of $\mathbb{F}_q$, is $\rho = (q^k - 1)/(q - 1)$. In addition, this set of characterizations will be useful for the present work, because it will allow us to present