

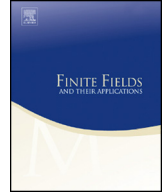


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A characterization of translation ovals in finite even order planes



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ABSTRACT

In this article we consider a set \mathcal{C} of points in $\text{PG}(4, q)$, q even, satisfying certain combinatorial properties with respect to the planes of $\text{PG}(4, q)$. We show that there is a regular spread in the hyperplane at infinity, such that in the corresponding Bruck–Bose plane $\text{PG}(2, q^2)$, the points corresponding to \mathcal{C} form a translation hyperoval, and conversely.

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1. Introduction

In this article we first consider a non-degenerate conic in $\text{PG}(2, q^2)$, q even. We look at the corresponding point set in the Bruck–Bose representation in $\text{PG}(4, q)$, and study its combinatorial properties (details of the Bruck–Bose representation are given in Section 2). Some properties of this set were investigated in [4]. In this article we are interested

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in combinatorial properties relating to planes of $\text{PG}(4, q)$. We consider a set of points in $\text{PG}(4, q)$ satisfying certain of these combinatorial properties and find that the points correspond to a translation oval in the Bruck–Bose plane $\text{PG}(2, q^2)$.

In [3], the case when q is odd is considered, and we show that given a set of points in $\text{PG}(4, q)$ satisfying the following combinatorial properties, we can reconstruct the conic in $\text{PG}(2, q^2)$. We use the following terminology in $\text{PG}(4, q)$: if the hyperplane at infinity is denoted Σ_∞ , then we call the points of $\text{PG}(4, q) \setminus \Sigma_\infty$ *affine points*.

Theorem 1.1. (See [3].) *Let Σ_∞ be the hyperplane at infinity in $\text{PG}(4, q)$, $q \geq 7$, q odd. Let \mathcal{C} be a set of q^2 affine points, called \mathcal{C} -points, and suppose there exists a set of planes called \mathcal{C} -planes satisfying the following properties:*

1. *Each \mathcal{C} -plane meets \mathcal{C} in a q -arc.*
2. *Any two distinct \mathcal{C} -points lie in a unique \mathcal{C} -plane.*
3. *The affine points of $\text{PG}(4, q)$ are of three types: points of \mathcal{C} ; points on no \mathcal{C} -plane; and points on exactly two \mathcal{C} -planes.*
4. *If a plane meets \mathcal{C} in more than four points, it is a \mathcal{C} -plane.*

Then there exists a unique spread \mathcal{S} in Σ_∞ so that in the Bruck–Bose translation plane $\mathcal{P}(\mathcal{S})$, the \mathcal{C} -points form a q^2 -arc of $\mathcal{P}(\mathcal{S})$. Moreover, the spread \mathcal{S} is regular, and so $\mathcal{P}(\mathcal{S}) \cong \text{PG}(2, q^2)$, and the q^2 -arc can be completed to a conic of $\text{PG}(2, q^2)$.

The case when q is even is more complex. The combinatorial properties only allow us to reconstruct a translation oval in $\text{PG}(2, q^2)$. The main result of this article is the following theorem.

Theorem 1.2. *Consider $\text{PG}(4, q)$, q even, $q > 2$, with the hyperplane at infinity denoted by Σ_∞ . Let \mathcal{C} be a set of q^2 affine points, called \mathcal{C} -points and consider a set of planes called \mathcal{C} -planes which satisfies the following:*

- (A1) *Each \mathcal{C} -plane meets \mathcal{C} in a q -arc.*
- (A2) *Any two distinct \mathcal{C} -points lie in a unique \mathcal{C} -plane.*
- (A3) *The affine points that are not in \mathcal{C} lie on exactly one \mathcal{C} -plane.*
- (A4) *Every plane which meets \mathcal{C} in at least three points either meets \mathcal{C} in exactly four points or is a \mathcal{C} -plane.*

Then there exists a regular spread \mathcal{S} in Σ_∞ such that in the Bruck–Bose plane $\mathcal{P}(\mathcal{S}) \cong \text{PG}(2, q^2)$, the \mathcal{C} -points, together with two extra points on ℓ_∞ , form a translation hyper-oval of $\text{PG}(2, q^2)$.

We begin in Section 2 with the necessary background material on the Bruck–Bose representation. In Section 3 we investigate combinatorial properties of conics and translation

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