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Existence of canonically inherited arcs in Moulton planes of odd order ☆

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We address the problem of determining the spectrum of all possible values of k for canonically inherited k-arcs in Moulton planes of odd order q^2, with $q \equiv 3 \pmod 4$, arising from hyperbole in $\mathrm{AG}(2, q^2)$.

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1. Introduction

An arc of size k (briefly, a k-arc) in a finite projective plane π of order $q = p^h$ is a set consisting of k points of π with the property that no three of them are collinear. A k-arc is said to be complete if it is not contained in a $(k+1)$-arc. Throughout this paper q is an odd prime power.

The upper bound on the number of points a k-arc can contain is $q+1$. A $(q+1)$-arc is called an oval, and examples of ovals in the Desarguesian projective plane $\mathrm{PG}(2,q)$ are the irreducible conics; see [13].

Concerning complete k-arcs which are different from ovals, the following upper bounds on the number of points are known; see for instance [6,7,9,13–15,33–36].

- $k \le q - \frac{\sqrt{q}}{2} + \frac{5}{2}$;
- $k \le \frac{44}{45}q + \frac{8}{9}$ if q is a prime.

Arcs different from conics, but sharing as many points as possible with conics, were also studied in [5,14,22,23].

Currently, k-arcs in the Desarguesian projective plane $\mathrm{PG}(2,q)$ have been investigated, not only from a theoretical point of view, but also for their natural applications to coding theory and cryptography; see for instance [11,12,17,18,21,27,28,32].

No q-arc in $\mathrm{PG}(2,q)$ can be complete; this fact was proven by B. Segre [25,26] for q odd, and by G. Tallini [31] for q even. Further, it has been conjectured that in $\mathrm{PG}(2,q)$ no $(q-1)$-arc is complete for $q > 13$. This problem has been recently addressed by M. Giulietti and G. Korchmáros who showed that if a complete $(q-1)$-arc in $\mathrm{PG}(2,q)$ with $q > 13$ exists, then it must have some particular geometrical features concerning intersection with conics and automorphisms; see [10, Section 7].

In [4] constructions of complete arcs in $\mathrm{PG}(2,q)$ are proposed. These constructions form families of k-arcs of all sizes k in an interval $k_{\min} \le k \le k_{\max}$ where $k_{\min}$ is of order $\frac{1}{3}q$ or $\frac{1}{4}q$ while $k_{\max}$ has order $\frac{1}{2}q$.

However, these results are not true in general, when the projective plane could be non-Desarguesian. G. Menichetti [24] constructed an infinite family of complete q-arcs in Hall planes of even order. R.H.F. Denniston [8] and A. Barlotti [3] exhibited examples of complete 9-arcs in non-Desarguesian projective planes of order 9. T. Szőnyi [29] provided constructions for complete $(q-1)$-arcs in the Hall plane of odd order $q > 49$ and for complete arcs having $k \ge \frac{2}{3}q$ points in André planes of square order. Further, T. Szőnyi [30] showed that the set of rational numbers $\frac{k}{q}$ such that there exists a complete k-arc in some projective plane of order q is dense in the interval $[0,1]$.

Generally speaking, every affine plane α of order $q = p^h$ can be regarded as an alteration of the Desarguesian plane $\mathrm{AG}(2,q)$. Points and lines of α are usually identified with those of $\mathrm{AG}(2,q)$, but some point–line incidences in $\mathrm{AG}(2,q)$ are altered in order to obtain the point–line incidences of α.