Finite Fields and Their Applications 37 (2016) 54–71

Contents lists available at ScienceDirect

Finite Fields and Their Applications

www.elsevier.com/locate/ffa

Necessary conditions for reversed Dickson polynomials of the second kind to be permutational ☆

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ARTICLE INFO

Article history:
Received 8 April 2014
Received in revised form 15 July 2015
Accepted 3 September 2015
Available online 25 September 2015
Communicated by Xiang-dong Hou

MSC:
11T06
11T55
11C08

Keywords:
Permutation polynomial
Reversed Dickson polynomial of the second kind

ABSTRACT

In this paper, we present several necessary conditions for the reversed Dickson polynomial $E_n(1, x)$ of the second kind to be a permutation of $\mathbb{F}_q$. In particular, we give explicit evaluation of the sum $\sum_{a\in\mathbb{F}_q} E_n(1, a)$.

☆ The research was supported partially by National Science Foundation of China Grant #11371260, Science and Technology Research Projects of Chongqing Education Committee Grant #KJ15012004, Research Projects of Yangtze Normal University Grant #2014KYQD04 and Science and Technology Plan Projects of Fuling Grant #FLKJ2015ABA1031.

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http://dx.doi.org/10.1016/j.ffa.2015.09.001
1071-5797/

Finite field
Generating function

1. Introduction

Let p be a prime and $\mathbb{F}_q$ be a finite field of $q = p^e$ elements, where e is a positive integer. Associated to any integer $n \geq 0$ and a parameter $a \in \mathbb{F}_q$, the n-th *Dickson polynomials of the first kind and of the second kind*, denoted by $D_n(x, a)$ and $E_n(x, a)$, are defined by

$$D_n(x,a) := \sum_{i=0}^{[\frac{n}{2}]} \frac{n}{n-i} \binom{n-i}{i} (-a)^i x^{n-2i}$$

and

$$E_n(x,a) := \sum_{i=0}^{[\frac{n}{2}]} \binom{n-i}{i} (-a)^i x^{n-2i},$$

respectively. Recently, Wang and Yucas [5] further defined the *n-th Dickson polynomial of the $(k+1)$-th kind* $D_{n,k}(x, a) \in \mathbb{F}_q[x]$ by

$$D_{n,k}(x,a) := \sum_{i=0}^{[\frac{n}{2}]} \frac{n-ki}{n-i} \binom{n-i}{i} (-a)^i x^{n-2i}.$$

On the other hand, Hou, Mullen, Sellers and Yucas [3] introduced the definition of *the reversed Dickson polynomial of the first kind*, denoted by $D_n(a, x)$, as follows

$$D_n(a,x) := \sum_{i=0}^{[\frac{n}{2}]} \frac{n}{n-i} \binom{n-i}{i} (-x)^i a^{n-2i}.$$

By extending the definition of reversed Dickson polynomials, Wang and Yucas [5] got the definition of *the* n-th *reversed Dickson polynomial of the* $(k+1)$-th *kind* $D_{n,k}(a, x) \in \mathbb{F}_q[x]$, which is defined by

$$D_{n,k}(a,x) := \sum_{i=0}^{[\frac{n}{2}]} \frac{n-ki}{n-i} \binom{n-i}{i} (-x)^i a^{n-2i}.$$

The permutation behavior of Dickson polynomials $D_n(x, a)$ over finite fields are well known: $D_n(x, 0) = x^n$ is a permutation polynomial of $\mathbb{F}_q$ if and only if $(n, q-1) = 1$, and if $a \neq 0$, then $D_n(x, a)$ induces a permutation of $\mathbb{F}_q$ if and only if $(n, q^2-1) = 1$ (see [4], Theorem 7.16). Meanwhile, there are many results on permutation properties of Dickson