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Rudnick and Soundararajan's theorem for function fields

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ABSTRACT

In this paper we prove a function field version of a theorem by Rudnick and Soundararajan about lower bounds for moments of quadratic Dirichlet L-functions. We establish lower bounds for the moments of quadratic Dirichlet L-functions associated to hyperelliptic curves of genus g over a fixed finite field $\mathbb{F}_q$ in the large genus g limit.

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1. Introduction

It is a fundamental problem in analytic number theory to estimate moments of central values of L-functions in families. For example, in the case of the Riemann zeta function the question is to establish asymptotic formulas for

$$M_k(T) := \int_1^T |\zeta(\tfrac{1}{2} + it)|^{2k} dt, \tag{1.1}$$

where k is a positive integer and $T \to \infty$.

A believed folklore conjecture asserts that, as $T \to \infty$, there is a positive constant C_k such that

$$M_k(T) \sim C_k T (\log T)^{k^2}. \tag{1.2}$$

Due to the work of Conrey and Ghosh [3] the conjecture above assumes a more explicit form, namely

$$C_k = \frac{a_k g_k}{\Gamma(k^2 + 1)}, \tag{1.3}$$

where

$$a_k = \prod_{p \text{ prime}} \left[\left(1 - \frac{1}{p}\right)^{k^2} \sum_{m \geq 0} \frac{d_k(m)^2}{p^m} \right], \tag{1.4}$$

g_k is an integer when k is an integer and $d_k(n)$ is the number of ways to represent n as a product of k factors.

Asymptotics for $M_k(T)$ are only known for $k = 1$, due to Hardy and Littlewood [7]

$$M_1(T) \sim T \log T, \tag{1.5}$$

and for $k = 2$, due to Ingham [10]

$$M_2(T) \sim \frac{1}{2\pi^2} T \log^4 T. \tag{1.6}$$

Unfortunately the recent technology does not allow us to obtain asymptotics for higher moments of the Riemann zeta function. The same statement applies for the higher moments of other L-functions. However, due to the precursor work of Keating and Snaith [14,15] and, subsequently, due to the work of Conrey, Farmer, Keating, Rubinstein and Snaith [4], and Diaconu, Goldfeld and Hoffstein [5], there are now very elegant conjectures for moments of L-functions.