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Multizeta shuffle relations for function fields with non rational infinite place ☆

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In contrast to the 'universal' multizeta shuffle relations, when the chosen infinite place of the function field over $\mathbb{F}_q$ is rational, we show that in the non-rational case, only certain interesting shuffle relations survive, and the $\mathbb{F}_q$-linear span of the multizeta values does not form an algebra. This is due to the subtle interactions between the larger finite field $\mathbb{F}_\infty$, the residue field of the completion at infinity where the signs live and $\mathbb{F}_q$, the field of constants where the coefficients live. We study the classification of these special relations which survive.

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1. Introduction

In 1775, more than 30 years after Euler introduced the zeta values $\zeta(s)$, he introduced and studied the multizeta values $\zeta(s_1, \cdots, s_r)$. They have resurfaced with renewed interest because of their connections with several other parts (see e.g., introduction of [8] for references) of mathematics, for example, in the Grothendieck–Ihara program to study the absolute Galois group of $\mathbb{Q}$ through the algebraic fundamental group of the projective line minus three points. Thus the understanding of the structure of relations between them is quite important. The simple trichotomy $n_i > n_{i+1}$ or $n_i = n_{i+1}$ or $n_i < n_{i+1}$ applied to the sum definition of multizeta (which is over such ordered tuples of natural numbers) shows (the so-called sum shuffle relations) that the product of two multizeta values is a linear combination of multizeta values, and thus the $\mathbb{Q}$-span of the multizeta values is an algebra.

While the trichotomy approach and these sum shuffle relations completely fail for the multizeta values of [6, Section 5.10], [8,9,2] for function fields over $\mathbb{F}_q$, it was shown in [9] that a different mechanism leads to a different kind of combinatorially involved shuffle identities (Theorem 3.1.1), which are 'universal' in the sense that they work for any function field together with a rational (i.e., degree one) place at infinity corresponding to the ring of integers A. In particular, the $\mathbb{F}_q$-span of all multizeta values is an algebra in this case.

We focus on this aspect, but point out in passing that, as in the classical case, these multizeta values also have connections with the absolute Galois group through the analogue of Ihara power series, in a work by G. Anderson with the second author, and with the periods of Carlitz–Tate–Anderson mixed t-motives [1,2], at least in the simplest case $A = \mathbb{F}_q[t]$.

In this paper, we look at the shuffle relations in the case where the place at infinity is not rational, so that there are more choices of signs (in the finite residue field of the completion) than those available in the finite residue field of the function field. This leads to two different natural approaches to define multizeta, and we show that in each approach, certain kind of (different for the two approaches) fundamental relations (Theorems 3.3.1, 3.4.1) survive! In Section 4, we discuss the results and conjectures on the classification of these relations, and in the last section we briefly mention higher depth situation.

The interesting form of the surviving relations as well as the numerical experimentation, admittedly quite limited, which suggests that these might be the only ones which survive, make us wonder if there is any deeper reason behind this.

2. Notation, background and definitions

$\mathbb{Z}$	{integers}
$\mathbb{Z}_+$	{positive integers}
q	a power of a prime p

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