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New strongly regular decompositions of the complete graphs with prime power vertices

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We construct two infinite families of strongly regular decompositions of complete graphs on finite fields, which consist of four negative Latin square type Cayley graphs. Our construction is based on the existence of cyclotomic strongly regular graphs.

1. Introduction

In this paper, we study a construction of strongly regular decompositions of a complete graph, i.e., decompositions of the edge set of a complete graph into spanning subgraphs

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that are all strongly regular. We will assume that the reader is familiar with the theories of strongly regular graphs and association schemes. For background on strongly regular graphs, we refer the reader to [5] and [7]. For the theory of association schemes, our main references are [1] and [12].

Let Γ be a simple and undirected graph and A be its adjacency matrix. A useful way to check whether Γ is strongly regular is by using the eigenvalues of A (which are usually called eigenvalues of Γ). For convenience, we will call an eigenvalue of Γ *restricted* if it has an eigenvector which is not a multiple of the all-ones vector. For a k-regular connected graph, the restricted eigenvalues are simply the eigenvalues different from k.

Theorem 1. *(See [5, Theorem 9.12].) For a simple k-regular graph Γ of order v, not complete or edgeless, with adjacency matrix A, the following are equivalent:*

(1) *Γ is strongly regular with parameters (v, k, λ, μ) for certain integers λ, μ,*
(2) *$A^2 = (\lambda - \mu)A + (k - \mu)I + \mu J$ for certain real numbers λ, μ, where I, J are the identity matrix and the all-ones matrix, respectively,*
(3) *A has precisely two distinct restricted eigenvalues.*

A strongly regular graph (SRG) is said to be of *Latin square type* (respectively, *negative Latin square type*) if $(v, k, \lambda, \mu) = (n^2, r(n - \epsilon), \epsilon n + r^2 - 3\epsilon r, r^2 - \epsilon r)$ and $\epsilon = 1$ (respectively, $\epsilon = -1$).

One of the most useful methods for constructing SRGs is by the Cayley graph construction. Let G be an additively written group of order v, and let D be a subset of G such that $0 \notin D$ and $-D = D$, where $-D = \{-d \mid d \in D\}$. The *Cayley graph on G with connection set D*, denoted by $Cay(G, D)$, is the graph with the elements of G as vertices; two vertices are adjacent if and only if their difference belongs to D. The survey of Ma [17] contains much of what is known about strongly regular Cayley graphs.

Since the complement of an SRG is also strongly regular, these form a strongly regular decomposition of a complete graph consisting of two graphs. Our main goal in this paper is to find new infinite families of strongly regular decompositions consisting of four Cayley graphs on finite fields, each of which has negative Latin square type parameters. Furthermore, any union of the four graphs also forms an SRG. In the language of association schemes, such a strongly regular decomposition forms an amorphous four-class association scheme. (More precisely, if any union of nontrivial relations of an association scheme again forms an association scheme, the original association scheme is called *amorphous*.) Such an association scheme has the property that all of its graphs are strongly regular. Moreover, they are all of Latin square type or negative Latin square type. On the other hand, van Dam [8] showed that the converse of this claim is also correct generalizing a result by Ito et al. [14], who showed that any association scheme consisting of (negative) Latin square type graphs is amorphous. See [8] and [9] for basic properties and known constructions of amorphous association schemes.