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Cyclic codes over the ring $\mathbb{Z}_p[u,v]/\langle u^2, v^2, uv - vu\rangle$

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ABSTRACT

Let p be a prime number. In this paper, we study cyclic codes over the ring $\mathbb{Z}_p[u,v]/\langle u^2, v^2, uv - vu\rangle$. We find a unique set of generators for these codes. We also study the rank and the Hamming distance of these codes. We obtain all except one ternary optimal code of length 12 as the Gray image of the cyclic codes over the ring $\mathbb{Z}_p[u,v]/\langle u^2, v^2, uv - vu\rangle$.

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1. Introduction

Let R be a ring. A linear code of length n over the ring R is an R-submodule of R^n. A linear code C of length n over the ring R is cyclic if $(c_{n-1}, c_0, \ldots, c_{n-2}) \in C$ whenever

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$(c_0, c_1, \ldots, c_{n-1}) \in C$. We can consider a cyclic code C of length n over the ring R as an ideal in the ring $R[x]/\langle x^n - 1\rangle$ via the following correspondence

$$R^n \to R[x]/\langle x^n - 1\rangle, \quad (c_0, c_1, \ldots, c_{n-1}) \mapsto c_0 + c_1 x + \cdots + c_{n-1}x^{n-1}.$$

The cyclic codes form an important class of codes due to their rich algebraic structures and play a significant role in algebraic coding theory. The structures of cyclic codes over rings have been discussed in a series of papers [1,2,5–9,13,16,19]. The structures of cyclic codes of length n over a finite chain ring have been discussed in [11], when n is not divisible by the characteristic of the residue field $\overline{R}$. The structures of cyclic codes of length n, where n is divisible by the characteristic of the residue field $\overline{R}$, over some finite chain ring have been discussed in [3,4,10,17].

Yildiz and Karadeniz in [20] have considered the ring $\mathbb{F}_2[u, v]/\langle u^2, v^2, uv - vu\rangle$, which is not a chain ring, and studied cyclic codes of odd length over that. They have found some good binary codes as the Gray images of these cyclic codes. The authors of [12] have considered the more general ring $\mathbb{F}_2[u_1, u_2, \cdots, u_k]/\langle u_i^2, u_j^2, u_i u_j - u_j u_i\rangle$, and studied the general properties of cyclic codes over these rings and characterized the nontrivial one-generator cyclic codes. Sobhani and Molakarimi in [18] extended these studies to cyclic codes over the ring $\mathbb{F}_{2^m}[u, v]/\langle u^2, v^2, uv - vu\rangle$.

Let $R_{u^2,v^2,p} = \mathbb{Z}_p + u\mathbb{Z}_p + v\mathbb{Z}_p + uv\mathbb{Z}_p$, where $u^2 = 0$, $v^2 = 0$, $uv = vu$ and p be a prime number. Note that the ring $R_{u^2,v^2,p}$ can also be viewed as the quotient ring $\mathbb{Z}_p[u, v]/\langle u^2, v^2, uv - vu\rangle$. In this paper, we discuss the structure of cyclic codes of arbitrary length over the ring $R_{u^2,v^2,p}$. We find a unique set of generators, rank and a minimal spanning set for these codes. We also find the Hamming distance of these codes for length p^l. Note that the rank and the Hamming distance of these cyclic codes for $p = 2$ have not been discussed in [12,18,20]. Hence getting the rank and the Hamming distance of the cyclic codes over $R_{u^2,v^2,p}$ is new even for $p = 2$. We obtain all except one optimal code over $\mathbb{Z}_3$ of length 12 as the images of cyclic codes over the ring $R_{u^2,v^2,3}$ under the Gray map. We also obtain some good ternary codes of length 36 as the Gray images of cyclic codes over the ring $R_{u^2,v^2,3}$.

Let $R_{u^2,p} = \mathbb{Z}_p + u\mathbb{Z}_p$, $u^2 = 0$ and $R_{u^2,p,n} = R_{u^2,p}[x]/\langle x^n - 1\rangle$. The techniques we have used to find a set of generators are similar to those in [3,17]. Let C be a cyclic code over the ring $R_{u^2,v^2,p}$. The idea to find a set of generators is as follows. We view the cyclic code C as an ideal in the ring $R_{u^2,v^2,p,n} = R_{u^2,v^2,p}[x]/\langle x^n - 1\rangle$. Then we define the projection map from $R_{u^2,v^2,p,n} \to R_{u^2,p,n}$ and we get an ideal in the ring $R_{u^2,p,n}$, which gives a cyclic code over the ring $R_{u^2,p}$. The structure of cyclic codes over the ring $R_{u^2,p}$ is known from [17]. By pullback, we find a set of generators for a cyclic code over the ring $R_{u^2,v^2,p}$. By using the division algorithm and direct computations, we find the rank of these cyclic codes. Again, the techniques we have used to find the minimum distance are similar to those in [3,17].

This paper is organized as follows. In Section 2, we discuss the preliminaries that we need. In Section 3 we give a unique set of generators for the cyclic codes C over the

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