Finite Fields and Their Applications 34 (2015) 250–264

Contents lists available at ScienceDirect

Finite Fields and Their Applications

www.elsevier.com/locate/ffa

The isotopism problem of a class of 6-dimensional rank 2 semifields and its solution

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A R T I C L E I N F O

Article history:
Received 17 April 2014
Received in revised form 13 January 2015
Accepted 20 January 2015
Available online 2 March 2015
Communicated by L. Storme

MSC:
12K10
51A40
51E99

Keywords:
Semifield
Semifield spread
Linear set

A B S T R A C T

In [5] three classes of rank two presemifields of order q^{2n}, with q and n odd, were exhibited, leaving as an open problem the isotopy issue. In [18], the authors faced with this problem answering the question whether these presemifields are new for $n > 3$. In this paper we complete the study solving the case $n = 3$.

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http://dx.doi.org/10.1016/j.ffa.2015.01.007
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1. Introduction

A finite *presemifield* $\mathbb{S} = (\mathbb{S}, +, \cdot)$ is a finite algebra satisfying all the axioms for a *skewfield* except the associativity for the multiplication and the existence of a multiplicative identity. If a presemifield has an identity element then it is called a *semifield*. Semifields have received a lot of attention in recent years, and we refer to [20] and [19] for references, general theory, and definitions, if they are not included here. A finite presemifield has order a power of a prime p and such a prime is called the *characteristic* of $\mathbb{S}$. Two presemifields $\mathbb{S} = (\mathbb{S}, +, \cdot)$ and $\mathbb{S}' = (\mathbb{S}', +, \circ)$ of characteristic p are said to be *isotopic* if there exist three $\mathbb{F}_p$-linear maps g_1, g_2 and g_3 from $\mathbb{S}$ into $\mathbb{S}'$ such that $g_1(x \cdot y) = g_2(x) \circ g_3(y)$ for all $x, y \in \mathbb{S}$. From the isotopy point of view, presemifields are not more general than semifields; in fact, it is easy to see that the isotopy class of a presemifield always contains a semifield [15, Theorem 4.5.4]. The definition of the *nuclei* and of the *center* of a semifield can be found, e.g., in [20]. If two (pre)semifields are isotopic their dimensions over the nuclei and over the center are invariant, and we refer to them as the *parameters* of the (pre)semifield. In [5] the author introduced three families of rank 2 presemifields of order q^n, with q and n odd, 2-dimensional over their left nucleus and $2n$-dimensional over their center. These presemifields are obtained starting from a pair of bijective $\mathbb{F}_q$-linear maps of $\mathbb{F}_{q^n}$, satisfying suitable conditions [5, Theorem 4.1], and they have been labeled $\mathcal{D}_A$, $\mathcal{D}_B$ and $\mathcal{D}_{AB}$ in [18]. Also, in [5, Theorem 4.3], the author determined their parameters. This information was insufficient to address the isotopy issue which, in fact, was leaved as an open problem. In [18] this problem was solved for $n > 3$, proving that presemifields in the families $\mathcal{D}_A$ and $\mathcal{D}_{AB}$ are new, i.e. not isotopic to any previously known semifield; whereas presemifields in the family $\mathcal{D}_B$ are isotopic to Generalized Twisted Fields for all $n \geq 3$. The remaining part of the case $n = 3$ is treated here separately; this mainly because in the relevant case there are many more known examples in the literature to compare with [2,6–9,14,12,13,15,17,29]. For $n = 3$ the Dempwolff presemifields $\mathcal{D}_A$ and $\mathcal{D}_{AB}$ are the algebraic structures $\mathcal{D}_A = (\mathbb{F}_{q^3} \times \mathbb{F}_{q^3}, +, \star_A)$ and $\mathcal{D}_{AB} = (\mathbb{F}_{q^3} \times \mathbb{F}_{q^3}, +, \star_{AB})$, q odd, having multiplications defined as follows

$$(u, v) \star_A (x, y) = (u, v) \begin{pmatrix} x & y \\ A_{a,r}(y) & \xi A_{a,r}(x) \end{pmatrix},$$

where $A_{a,r}(x) = x^{q^r} - a x^{q^{-r}}$, $r \in \{1, 2\}$, such that ξ is a nonsquare in $\mathbb{F}_q$ and $a \in \mathbb{F}_{q^3}^*$ with $N_{q^3/q}(a) \neq 1$,[1] and

$$(u, v) \star_{AB} (x, y) = (u, v) \begin{pmatrix} x & y \\ A_{b^2,r}(y) & \xi B_{b,-r}(x) \end{pmatrix},$$

[1] $N_{q^3/q}$ denotes the norm function of $\mathbb{F}_{q^3}$ over $\mathbb{F}_q$.